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Antenna Gain Loss and Pattern Degradation
due to
Transmission Through Dielectric Radomes

by

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Antenna Gain Loss and Pattern Degradation due to Transmission Through Dielectric Radomes

by

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ABSTRACT

A computer model for axially symmetric dielectric radomes in the near field of a circular aperture has been developed. This model, based on a method of moments solution for bodies of revolution, is used to determine the electric field pattern of a linearly polarized circular aperture radiating through a dielectric radome. The program is written in FORTRAN and can accommodate rotationally symmetric radome shapes. Arbitrary illumination distributions for the aperture can be specified, and phased scanning of the aperture in both azimuth and elevation is allowed. Computational run time has been reduced by taking advantage of mode symmetry. A separate program has been developed to compute gain, and together these programs are used to determine the pattern degradation and gain loss due to the presence of a radome in the near field.

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I. INTRODUCTION

A radome is a structure designed to protect an antenna in its operating environment. Aircraft radomes must protect radar antennas from high aerodynamic forces while minimizing drag, weight, and interference on its electrical operation. The presence of a radome can affect the gain, beamwidth, sidelobe level, and direction of boresight, as well as change the VSWR and antenna noise temperature. This thesis involves the continued development of a computer code that predicts antenna radiation patterns looking through aerodynamic radomes.

The ability to accurately predict the effect of a radome on the operation of an antenna early in the design process is desirable. Typically, designers of the earliest radomes were concerned more with structural considerations than electrical performance. However, after the near failure of the U.S. Army's "Pathfinder" program of radar-equipped B-24 "Liberator" bombers due to poor electrical performance of their radomes, it became clear that," There was a serious need for developing sound theoretical procedure for the electrical design of radomes." [Ref. 1] For most practical radomes the surface geometry is too complex to yield closed-form mathematical expressions describing their performance. It is possible to predict radome performance using numerical solutions of Maxwell's equations. The program described in this thesis is based on a numerical technique called the method of moments (MM). The particular form of the solution used here is based on a formulation by Mautz and Harrington [Ref. 7] for bodies of revolution. Francis [Ref. 2] developed a computer code *LDBORMM.F* that specialized the Mautz and Harrington solution to ogive-

shaped radomes. There are no restrictions on the location of the radome with respect to the antenna, and therefore near-field radomes can be analyzed.

The goal of this thesis was to extend the capabilities of this program. Significant improvements include: taking advantage of symmetry to reduce computation time, applying realistic radar antenna patterns for comparison to experimental results, and the computation of gain loss due to the radome.

A. DESCRIPTION OF THE PROBLEM

The basic system analyzed is a radome consisting of a body of revolution (BOR) located coaxial with a circular antenna, as in Figure 1.1. The BOR constraint is satisfied by many aerodynamic radomes and greatly simplifies the MM solution , which will be described in more detail in following chapters. The

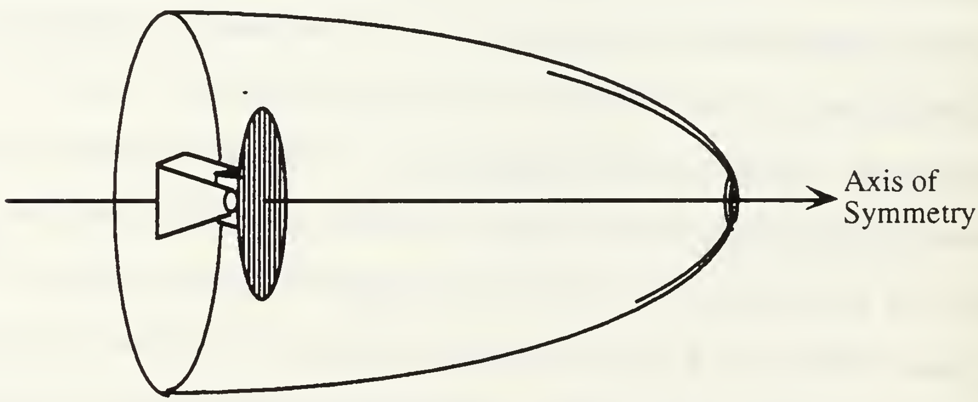

Figure 1.1 Radome and Antenna

ogive shape of the radome in Figure 1.1 is the primary one of interest, but the program can also model other bodies of revolution including: a cone, disk, hemisphere, and parabola, as shown in Figure 1.2. These were primarily added for validation purposes.

There are no constraints on the size, shape and location of these objects provided they are coaxial with the antenna. However, because MM requires the

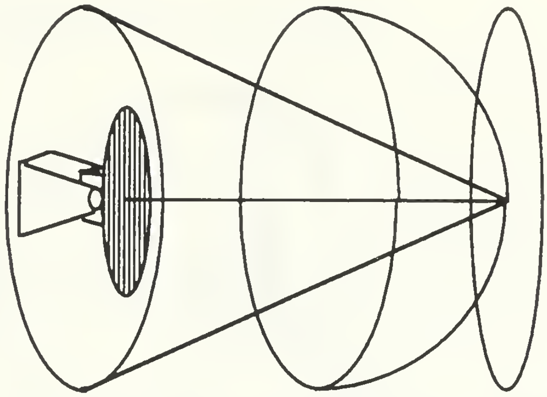

Figure 1.2 Optional Radome Shapes

solution of a matrix equation, computer run time and memory will limit the size of the bodies that can be analyzed. A unique feature of this program is that it takes into consideration the fact that the radome may be in the near field of the antenna as in Figure 1.3. The antenna aperture used in the program is circular, but the program can be easily altered to accommodate other shapes. There are no constraints on the aperture size, scan angle, or polarization.

The remainder of this thesis is divided into three major sections. Chapter II covers theoretical development of the problem and method of solution. The MM solution is described in Chapter II along with a discussion of mode symmetry and how it can be used to advantage in obtaining the radome current. Chapter II also describes how the total field is calculated from the currents. Chapter III covers the programs used for generating patterns based on the methods of

Chapter II. The first part describes the program *RADOME.F* which calculates the current and electric field. The second part describes the program *GAIN.F* which

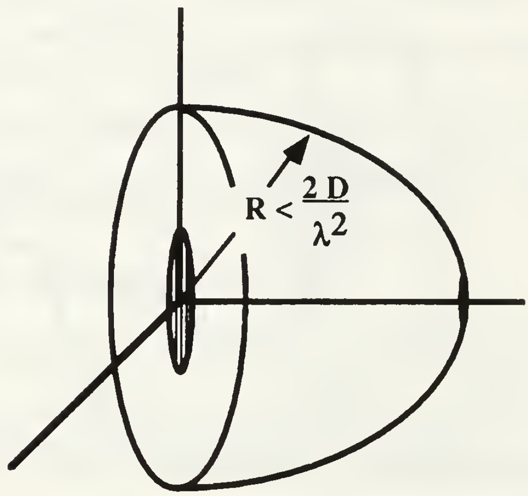

Figure 1.3 Radome Located in Near Field of Antenna

calculates the gain of the system. The final chapter discusses the results and presents several conclusions based on the calculated radome patterns.

II. ANALYSIS

A. GENERAL DESCRIPTION OF ANALYSIS

The objective of the analysis is to determine the radiation pattern of the antenna and its gain in the presence of a radome. In order to analyze the system, the spherical coordinates of Figure 2.1 are used. The total electric field in the direction of unit vectors $(\bar{\theta},\bar{\phi})$ is required to compute the gain in the far field. The total electric field is the sum of the electric field due to the currents on the antenna (radiated field), and the electric field due to the currents on the radome (scattered field). The procedure is to assume a current distribution on the antenna, then use the MM to determine currents induced on the radome. In practice, scattering from the radome back toward the antenna will cause a change in the initial current distribution. This effect will be small for a well-designed radome and is ignored in the analysis. Scattering back toward the antenna will primarily affect the VSWR looking into its terminals.

1. Determination of Gain

Gain is a measure of the ability of an antenna to concentrate radiated power in a particular direction. The maximum value of gain for a lossless antenna is the directivity G_o, which is the ratio of the maximum radiation intensity to the average radiation intensity [Ref. 3:p 610].

$$G_o = \frac{4\pi U(\theta,\phi)_{max}}{\oint\!\!\oint_s U(\theta,\phi)\,d\Omega}, \tag{2-1}$$

where $U(\theta, \phi)$ is the radiation intensity and $d\Omega = \sin\theta d\theta d\phi$ is the differential solid angle. Throughout this paper a lossless antenna will be assumed and the

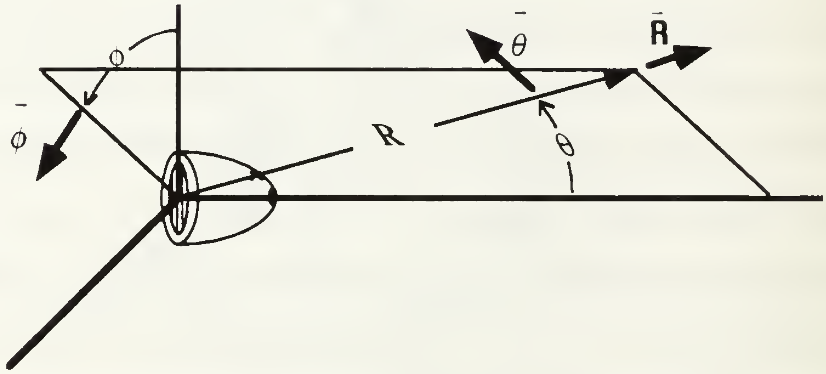

Figure 2.1 Coordinate System

terms gain and directivity used interchangeably.

Radiation intensity is the time-averaged power per unit solid angle, and is proportional to the magnitude of the electric field squared

$$U = \frac{1}{2\eta}\left|\bar{\mathbf{E}}\right|^2, \tag{2-2}$$

where

$$\left|\bar{\mathbf{E}}\right| = \left|\bar{\mathbf{E}}^{a}\right| + \left|\bar{\mathbf{E}}^{s}\right|. \qquad (2\text{-}3)$$

and $\bar{\mathbf{E}}^{a}$ is the electric field from the antenna, $\bar{\mathbf{E}}^{S}$ is the electric field scattered by the currents induced on the radome, and η is the impedance of free space.

2. Determining the Electric Field

The total electric field is calculated from the current distributions on the antenna and radome as illustrated in Figure 2.2. A current distribution $\bar{\mathbf{J}}_a$ on the antenna is assumed known. The currents induced on the radome are dependent on the current distribution on the antenna as shown in Figure 2.3, and this is taken into account in the method of moments solution.

Maxwell's equations, are used to determine the electric field at a point in space due to a current density $\bar{\mathbf{J}}$. When both the radome and antenna are present, the total current density is $\bar{\mathbf{J}} = \bar{\mathbf{J}}_a + \bar{\mathbf{J}}_s$. A solution of Maxwell's equations is

$$\bar{\mathbf{E}} = -j\omega\bar{\mathbf{A}} - \frac{j}{\omega\mu\varepsilon}\nabla\left(\nabla \bullet \bar{\mathbf{A}}\right), \qquad (2\text{-}4)$$

where

$$\bar{\mathbf{A}} = \frac{\mu}{4\pi}\iint \bar{\mathbf{J}}(r,\theta,\phi)\frac{e^{-jkr}}{r}ds', \qquad (2\text{-}5)$$

and

$$r = \left|\bar{\mathbf{R}} - \bar{\mathbf{R}}'\right|. \qquad (2\text{-}6)$$

In (2-4) $\bar{\mathbf{A}}$ is the vector potential, $\bar{\mathbf{R}}$ the position vector to the observation point and $\bar{\mathbf{R}}'$ the position vector to a source point on the antenna.

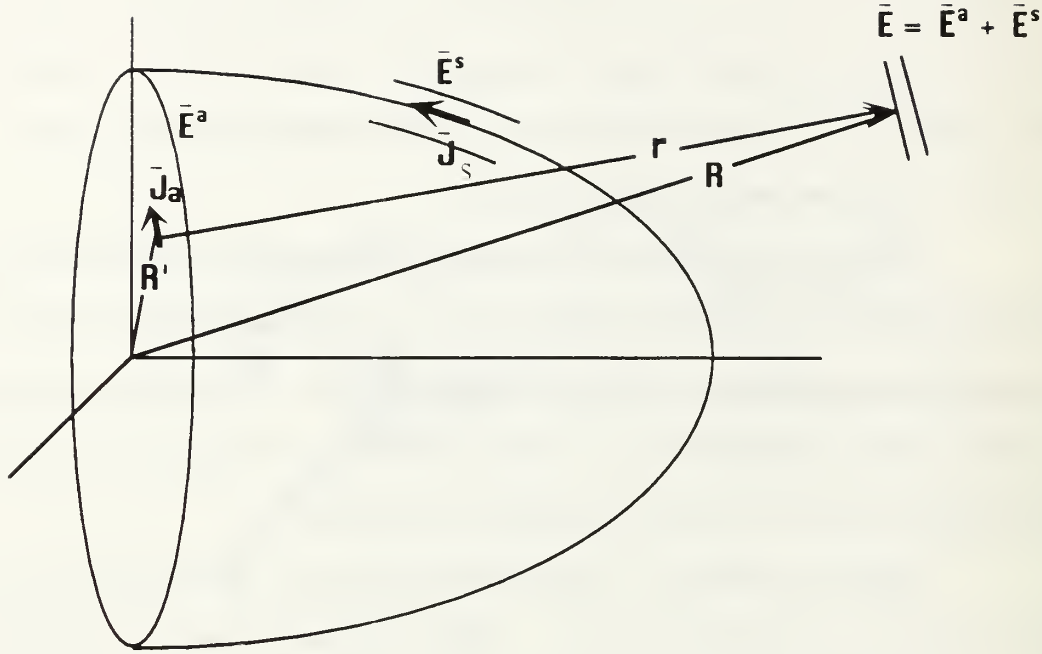

Figure 2.2 Electric Field from Currents on Antenna and Radome

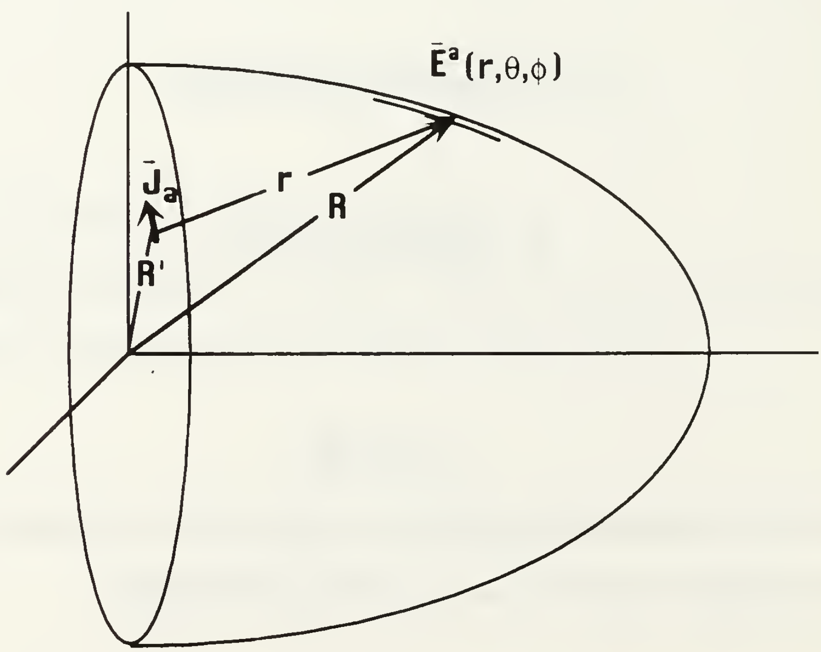

Figure 2.3 Electric Field Impinging on Radome

B. ANALYTICAL TECHNIQUE

Closed form mathematical solutions of the gain and radiation patterns are only possible for a limited number of applications that neglect the presence of the radome. In general, determining the radiation pattern and gain with the radome requires a numerical solution of Maxwell's equations. The radome geometry results in an unknown current distribution so (2-4) cannot be solved directly. However, boundary conditions can be applied on the radome surface which yield an integral equation for the current. The integral equation will be solved numerically using the method of moments.

1. E-field Integral Equation and the Method of Moments

The method of moments (MM) is a procedure used to solve an integral equation obtained by enforcing (2-4) on the radome surface [Ref. 5]. Equation (2-4) is used to determine the electric field $\bar{\mathbf{E}}^{a}$ incident upon the surface of the radome from the currents on the antenna $\bar{\mathbf{J}}_a$. Initially the surface will be assumed a perfect electric conductor (PEC), in which case the total $\bar{\mathbf{E}}$ tangent to the surface of the radome is equal to zero. Later a correction term will be added to the perfect conductor solution to account for the dielectric properties of the radome.

The boundary condition on the PEC radome is

$$\bar{\mathbf{E}}^{s}{}_{\tan} = -\bar{\mathbf{E}}^{a}{}_{\tan}. \tag{2-7}$$

As applied to (2-4),

$$\bar{\mathbf{E}}_{\tan}{}^{a} = j\omega\mu \iint_{s} \bar{\mathbf{J}}_{s} G\,ds + \frac{j}{\omega\varepsilon}\nabla\left[\nabla \bullet \iint_{s} \bar{\mathbf{J}}_{s} G\,ds\right], \tag{2-8}$$

where

$$G = \frac{e^{-jkr}}{4\pi r}, \tag{2-9}$$

and

$$r = \left|\bar{\mathbf{R}} - \bar{\mathbf{R}}'\right|. \tag{2-6}$$

Equation (2-8) is the E-field integral equation (EFIE). The only unknown is $\bar{\mathbf{J}}_S$, and once determined it is possible to calculate the scattered field by applying (2-4) to the radome.

To solve (2-8), the current $\bar{\mathbf{J}}_S$ is expanded into a series,

$$\bar{\mathbf{J}}_S = \sum_{i=1}^{N} I_i \bar{\mathbf{J}}_i. \tag{2-10}$$

The $\bar{\mathbf{J}}_i$ are basis functions, and the I_i are the expansion coefficients to be determined.

Mautz and Harrington applied this technique to the specific case of surfaces S generated by revolving a plane curve about the z-axis [Ref.5]. The plane curve is approximated by a series of straight-line segments connecting N_p points. The surface has a circular cross section in the azimuth variable ϕ, thus the body is represented by a series of "faceted rings" as shown in Figure 2.4.

The surface coordinate system is cylindrical (ρ, ϕ, z), and $\bar{\mathbf{t}}$ is a vector tangent to the surface and orthogonal to $\vec{\phi}$ at every point. In order for the series expansion of the current (2-10) to approximate an arbitrary surface current density $\bar{\mathbf{J}}_s$, independent sets of functions are defined as follows:

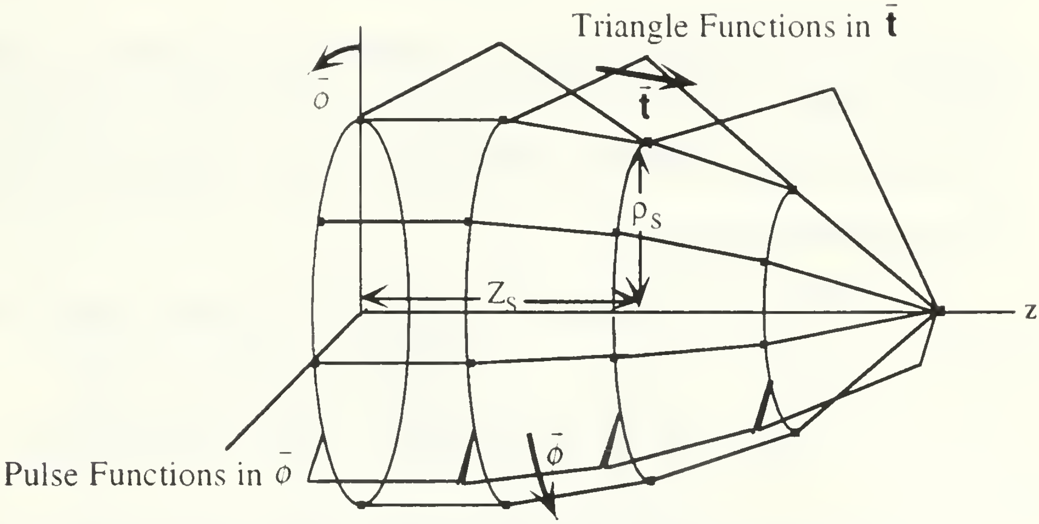

Figure 2.4 Segmented Radome with Current Basis Functions Depicted

$$\text{in } \bar{\mathbf{t}}: \quad \bar{\mathbf{J}}^{t}_{nj} = \bar{\mathbf{t}}\frac{T_j(t)}{\rho_s}e^{jn\phi} \qquad n = 0, \pm 1, \ldots, \pm\infty \qquad j = 1, 2, \ldots, N_p - 2$$

$$\text{in } \bar{\phi}: \quad \bar{\mathbf{J}}^{\phi}_{nj} = \bar{\phi}\frac{P_j(t)}{\rho_s}e^{jn\phi} \qquad n = 0, \pm 1, \ldots, \pm\infty \qquad j = 1, 2, \ldots, N_p - 1 \,. \quad (2\text{-}11)$$

$T_j(t)$ is a triangle function extending over two segments, and $P_j(t)$ is a pulse function extending over a single segment. For an explanation of these functions see [Ref. 1:pp 16-18]. Each value of n is referred to as an azimuthal

mode. The series expansion of (2-10) in terms of the chosen basis functions becomes,

$$\bar{\mathbf{J}}_s = \sum_{n=-\infty}^{\infty}\left[\sum_{j=1}^{Np-2} I^t_{nj}\bar{\mathbf{J}}^t_{nj} + \sum_{j=1}^{Np-1} I^\phi_{nj}\bar{\mathbf{J}}^\phi_{nj}\right]. \tag{2-12}$$

In order for the series to be an exact solution, the sum must be extended over infinite modes $-\infty \leq n \leq \infty$. Of course the series must be truncated, and the maximum number of modes for convergence n_{max} is dependent upon the antenna scan angle and radome size.

When the series expansion is applied to the integral equation (2-8),

$$\begin{aligned}
\bar{\mathbf{E}}^{a\ t}_{tan} &= \sum_{n=-\infty}^{+\infty}\sum_{j=1}^{Np-2} I^t_{nj}\iint_{S_i}\left[j\omega\mu\bar{\mathbf{J}}^t_{nj}G - \frac{-j}{\omega\varepsilon}\left(\nabla'\bullet\bar{\mathbf{J}}^t_{nj}\right)\nabla G\right]ds'.\\
\bar{\mathbf{E}}^{a\ \phi}_{tan} &= \sum_{n=-\infty}^{+\infty}\sum_{j=1}^{Np-1} I_{nj}\iint_{S_i}\left[j\omega\mu\bar{\mathbf{J}}^\phi_{nj}G - \frac{-j}{\omega\varepsilon}\left(\nabla'\bullet\bar{\mathbf{J}}^\phi_{nj}\right)\nabla G\right]ds'.
\end{aligned} \tag{2-13}$$

In order to solve for the unknowns I^t_{nj} and I^ϕ_{nj}, both sides of (2-13) are multiplied by testing functions $\bar{\mathcal{W}}^t_{mi}$ and $\bar{\mathcal{W}}^\phi_{mi}$, and integrated. Using Galerkin's Method [Ref. 8], the testing functions are chosen to be the complex conjugates of the basis functions $\bar{\mathbf{J}}^{t,\phi}_{nj}$,

$$\begin{aligned}
\bar{\mathcal{W}}^t_{mi} &= \bar{\mathbf{t}}\frac{T_i(t)}{\rho_s}e^{-jm\phi}\\
\bar{\mathcal{W}}^\phi_{mi} &= \bar{\phi}\frac{P_i(t)}{\rho_s}e^{-jm\phi}.
\end{aligned} \tag{2-14}$$

Multiplying both sides of (2-13) by each of the testing functions and integrating results in a set of $2N_p - 3$ simultaneous linear equations for each value of n. In matrix form, these equations are written as

$$[\mathbf{U}] = [\mathbf{Z}][\mathbf{I}], \tag{2-15}$$

where $[\mathbf{U}]$ is the excitation vector, $[\mathbf{Z}]$ is an impedance matrix and $[\mathbf{I}]$ contains the unknown current coefficients. Submatrices associated with the $\bar{\mathbf{t}}$, and $\bar{\phi}$ components (2-15) can be identified as follows:

$$\begin{bmatrix} [U_n^t] \\ [U_n^\phi] \end{bmatrix} = \begin{bmatrix} [Z_n^{tt}] & [Z_n^{t\phi}] \\ [Z_n^{\phi t}] & [Z_n^{\phi\phi}] \end{bmatrix} \begin{bmatrix} [I_n^t] \\ [I_n^\phi] \end{bmatrix} \qquad n = 0, \pm 1, \pm 2, \ldots, n_{max}. \tag{2-16}$$

The unknown coefficients are determined by solving the matrix equation

$$[\mathbf{I}] = [\mathbf{Z}]^{-1}[\mathbf{U}]. \tag{2-17}$$

The MM solution for the unknown current coefficients I_{nj}^t and I_{nj}^ϕ in vector form is

$$\begin{bmatrix} [I_n^t] \\ [I_n^\phi] \end{bmatrix} = \begin{bmatrix} [Z_n^{tt}] & [Z_n^{t\phi}] \\ [Z_n^{\phi t}] & [Z_n^{\phi\phi}] \end{bmatrix}^{-1} \begin{bmatrix} [U_n^t] \\ [U_n^\phi] \end{bmatrix}. \tag{2-18}$$

Once these are determined, it is possible to compute the electric field due to the current density distribution $\bar{\mathbf{J}}_s$ on the surface of the radome at any point in space.

The elements of each submatrix of the excitation vector $[\mathbf{U}]$ are

$$U^t_{ni} = \frac{1}{\eta}\iint_S \bar{W}^t_{ni} \bullet \bar{\mathbf{E}}^{a\,t}_{\tan} ds$$

$$U^\phi_{ni} = \frac{1}{\eta}\iint_S \bar{W}^\phi_{ni} \bullet \bar{\mathbf{E}}^{a\,\phi}_{\tan} ds, \tag{2-19}$$

and the elements of each of the submatrices of the impedance matrix $[\mathbf{Z}]$ are

$$\left[Z^{tt}_n\right]_{il} = \frac{1}{\eta}\iint_{s_l} \bar{W}^t_{nl} \bullet \iint_{s_i}\left[j\omega\mu\bar{\mathbf{J}}^t_{ni}G - \frac{-j}{\omega\varepsilon}\left(\nabla' \bullet \bar{\mathbf{J}}^t_{ni}\right)\nabla G\right]ds'ds$$

$$\left[Z^{t\phi}_n\right]_{il} = \frac{1}{\eta}\iint_{s_l} \bar{W}^t_{nl} \bullet \iint_{s_i}\left[j\omega\mu\bar{\mathbf{J}}^\phi_{ni}G - \frac{-j}{\omega\varepsilon}\left(\nabla' \bullet \bar{\mathbf{J}}^\phi_{ni}\right)\nabla G\right]ds'ds$$

$$\left[Z^{\phi t}_n\right]_{il} = \frac{1}{\eta}\iint_{s_l} \bar{W}^\phi_{nl} \bullet \iint_{s_i}\left[j\omega\mu\bar{\mathbf{J}}^t_{ni}G - \frac{-j}{\omega\varepsilon}\left(\nabla' \bullet \bar{\mathbf{J}}^t_{ni}\right)\nabla G\right]ds'ds$$

$$\left[Z^{\phi\phi}_n\right]_{il} = \frac{1}{\eta}\iint_{s_l} \bar{W}^\phi_{nl} \bullet \iint_{s_i}\left[j\omega\mu\bar{\mathbf{J}}^\phi_{ni}G - \frac{-j}{\omega\varepsilon}\left(\nabla' \bullet \bar{\mathbf{J}}^\phi_{ni}\right)\nabla G\right]ds'ds. \tag{2-20}$$

The fact that the testing functions $\bar{W}^{t,\phi}_{ml}$ are orthogonal in φ to $\bar{\mathbf{J}}^{t,\phi}_{ni}$ due to the presence of the exponentials has been used to eliminate elements of (2-20) with $n \neq m$ [Ref. 6]. This allows each mode n to be treated independently, thereby simplifying the matrix inversion problem. Thus, by constraining the surface to be rotationally symmetric, the azimuthal dependence of the current is expressed as a sum over exponentials (a Fourier series) and each mode n is independent.

a. Impedance of Thin-Shell Dielectric Radomes

The impedance matrix $[\mathbf{Z}]$ of the MM solution assumes that the

radome surface is a PEC, where (2-7) is applied as a boundary condition of (2-4). For thin-shell dielectric radomes, a correction term is added to the impedance matrix that allows the modeling of radomes of arbitrary dielectric constant [Ref.9].

When the electromagnetic wave from the antenna is incident on a dielectric radome that has an intrinsic impedance

$$\eta = \sqrt{\mu / \varepsilon} \tag{2-21}$$

different from that of free space (η_o), there will be reflection and refraction as shown in Figure 2.5 [Ref. 3:p. 397]. The total fields are

$$\begin{aligned} \bar{\mathbf{E}} &= \bar{\mathbf{E}}^a + \bar{\mathbf{E}}^s \\ \bar{\mathbf{H}} &= \bar{\mathbf{H}}^s + \bar{\mathbf{H}}^a . \end{aligned} \tag{2-22}$$

These fields satisfy Maxwell's equations

$$\nabla \times \left(\bar{\mathbf{E}}^a + \bar{\mathbf{E}}^s\right) = -j\omega\mu_o\left(\bar{\mathbf{H}}^a + \bar{\mathbf{H}}^s\right) \tag{2-23}$$

and,

$$\nabla \times \left(\bar{\mathbf{H}}^a + \bar{\mathbf{H}}^s\right) = j\omega\varepsilon_o\left(\bar{\mathbf{E}}^a + \bar{\mathbf{E}}^s\right) + j\omega\left(\varepsilon - \varepsilon_o\right)\left(\bar{\mathbf{E}}^a + \bar{\mathbf{E}}^s\right), \tag{2-24}$$

where the second term exists only when there is material present with a permittivity different than free space. This term is a polarization current $\bar{\mathbf{J}}$, where

$$\bar{\mathbf{J}} \equiv j\omega\left(\varepsilon - \varepsilon_o\right)\bar{\mathbf{E}} . \tag{2-25}$$

Maxwell's equations are also satisfied by the field from the antenna $\left(\bar{\mathbf{E}}^a, \bar{\mathbf{H}}^a\right)$. Since $\left(\bar{\mathbf{E}}, \bar{\mathbf{H}}\right)$ and $\left(\bar{\mathbf{E}}^a, \bar{\mathbf{H}}^a\right)$ satisfy Maxwell's equations, by superposition and (2-22) $\left(\bar{\mathbf{E}}^s, \bar{\mathbf{H}}^s\right)$ also satisfy Maxwell's equations. Therefore

$$\nabla \times \bar{\mathbf{H}}^s = j\omega \bar{\mathbf{E}}^s + \bar{\mathbf{J}}. \tag{2-26}$$

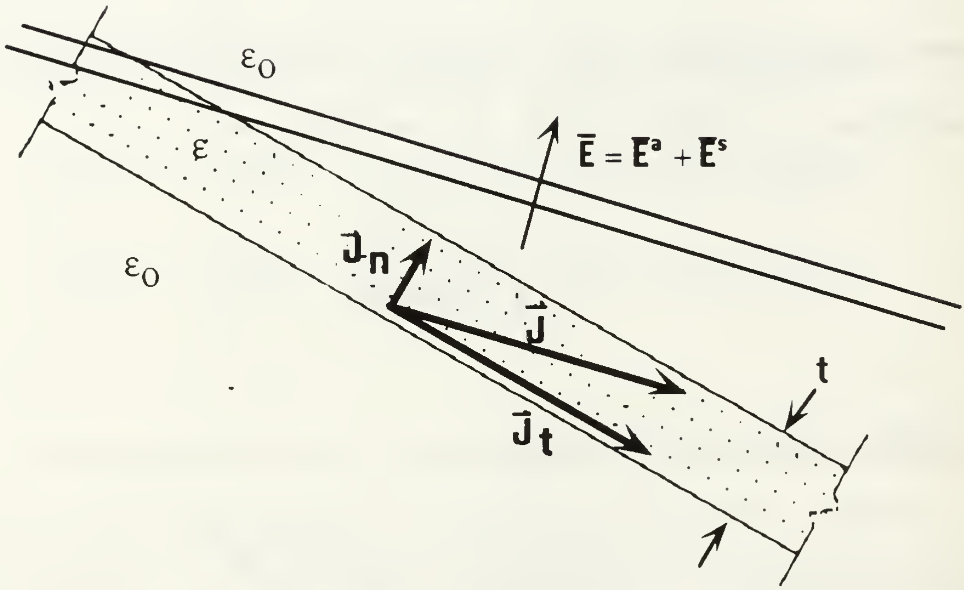

Figure 2.5 Discontinuity of Permittivity for Dielectric Radomes

For a radome consisting of a thin dielectric shell, the polarization current has a large tangential component and small normal component. If only the tangential component is significant, then (2-8) can be modified to include dielectric properties of the radome as follows:

$$\bar{\mathbf{E}}_{tan}{}^{a} = \frac{\bar{\mathbf{J}}_s}{j\omega(\varepsilon - \varepsilon_o)t} - j\omega\mu \iint_s \bar{\mathbf{J}}_s G ds + \frac{j}{\omega\varepsilon} \nabla \left[\nabla \bullet \iint_s \bar{\mathbf{J}}_s G ds \right]. \tag{2-27}$$

The first term on the right side is the impedance loading term; the integro-differential portion remains unchanged from the PEC assumption.

After multiplying both sides of (2-27) by the testing functions $\left(\bar{\boldsymbol{W}}^{t}_{mi}, \bar{\boldsymbol{W}}^{\phi}_{mi}\right)$ and integrating, the impedance matrix $\left[\mathbf{Z}\right]$ of (2-15) becomes

$$\left[\mathbf{Z}\right] = \left[\mathbf{Z}_{MM}\right] + \left[\mathbf{Z}_{L}\right], \tag{2-28}$$

where the elements of $\left[\mathbf{Z}_{MM}\right]$ are given by (2-20), and

$$\left[\mathbf{Z}^{tt}_{Ln}\right]_{ij} = \iint_s \bar{\boldsymbol{W}}^{t}_{i} \bullet \bar{\mathbf{J}}^{t}_{s\,jn} Z_L ds$$

$$\left[\mathbf{Z}^{t\phi}_{Ln}\right]_{ij} = \iint_s \bar{\boldsymbol{W}}^{t}_{i} \bullet \bar{\mathbf{J}}^{\phi}_{s\,jn} Z_L ds = \mathbf{0}, \text{ (by orthogonality of } \bar{\mathbf{t}}, \text{ and } \bar{\phi}\text{)}$$

$$\left[\mathbf{Z}^{\phi t}_{Ln}\right]_{ij} = \iint_s \bar{\boldsymbol{W}}^{\phi}_{i} \bullet \bar{\mathbf{J}}^{t}_{s\,jn} Z_L ds = \mathbf{0}, \text{ (by orthogonality of } \bar{\phi}, \text{ and } \bar{\mathbf{t}}\text{)}$$

$$\left[\mathbf{Z}^{\phi\phi}_{Ln}\right]_{ij} = \iint_s \bar{\boldsymbol{W}}^{\phi}_{i} \bullet \bar{\mathbf{J}}^{\phi}_{s\,jn} Z_L ds, \tag{2-29}$$

and

$$Z_L = \frac{1}{j\omega\left(\varepsilon - \varepsilon_o\right)t}. \tag{2-30}$$

The thickness t in the impedance loading term is necessary to convert the volume current density $\bar{\mathbf{J}}_t$ of Figure 2.5 to a surface current density $\bar{\mathbf{J}}_s$.

Solving (2-17) yields

$$\left[\mathbf{I}\right] = \left[\mathbf{Z}_{MM} + \mathbf{Z}_{L}\right]^{-1}\left[\mathbf{U}\right]. \tag{2-31}$$

The radome impedance Z_L can be rewritten in terms of dielectric constant ε_r, thickness to wavelength ratio n_λ, and the loss tangent $\tan\delta$ [Ref. 2:pp 45-46],

$$Z_L = \frac{60}{n_\lambda\left[j\left(\varepsilon_r - 1\right) + \varepsilon_r \tan\delta\right]}. \tag{2-32}$$

Note that both (2-29) and (2-32) allow the modeling of radomes whose permittivity may be complex, $\varepsilon = \varepsilon' - j\varepsilon''$.

2. Mode Symmetry

When the surface current density $\bar{\mathbf{J}}_s$ is expanded into a series as in (2-12), the sum is extended over azimuthal modes $n = 0, \pm 1, \pm 2, \cdots, \pm n_{max}$. The mode index appears in the exponential term of basis functions $\bar{\mathbf{J}}^{t\phi}_{nj}$ (2-11) and testing functions $\bar{\boldsymbol{W}}^{t\phi}_{nj}$ (2-14). The exponential factor can be written as,

$$e^{jn\phi} = \cos(n\phi) + j\sin(n\phi). \tag{2-33}$$

For each n there is a current vector $\mathbf{I}_n$ given by (2-18). Mode symmetry refers to a relationship between the positive and negative mode vectors $\mathbf{I}_n$ and $\mathbf{I}_{-n}$. If mode symmetry exists, then only one matrix equation needs to be evaluated rather than two. This greatly reduces the computational effort.

Recall the definition of the excitation vector $\left[\bar{\mathbf{U}}\right]$

$$\begin{aligned} U^t_{ni} &= \frac{1}{\eta}\iint_S \bar{\boldsymbol{W}}^t_{ni} \bullet \bar{\mathbf{E}}^{a\,t}_{\tan}\, ds \\ U^\phi_{ni} &= \frac{1}{\eta}\iint_S \bar{\boldsymbol{W}}^\phi_{ni} \bullet \bar{\mathbf{E}}^{a\,\phi}_{\tan}\, ds. \end{aligned} \tag{2-19}$$

The electric field from the antenna is

$$\bar{\mathbf{E}}^a = E^a_R(R,\theta,\phi)\bar{\mathbf{R}} + E^a_\theta(R,\theta,\phi)\bar{\theta} + E^a_\phi(R,\theta,\phi)\bar{\phi}, \tag{2-34}$$

which on the radome surface can be expressed in terms of the two orthogonal components $\bar{\boldsymbol{t}}$ and $\bar{\phi}$

$$\bar{\mathbf{E}}^a = E^a_t(t,\phi)\bar{\boldsymbol{t}} + E^a_\phi(t,\phi)\bar{\phi}. \tag{2-35}$$

In the far field of the antenna, the $\bar{\mathbf{R}}$ component in (2-34) decays to zero and E_t^a consists only of E_θ^a. Therefore as shown in Figure 2.6,

$$E_t^a(t,\phi) = E_t^a(t)\cos\phi$$
$$E_\phi^a(t,\phi) = E_\phi^a(t)\sin\phi. \qquad (2\text{-}36)$$

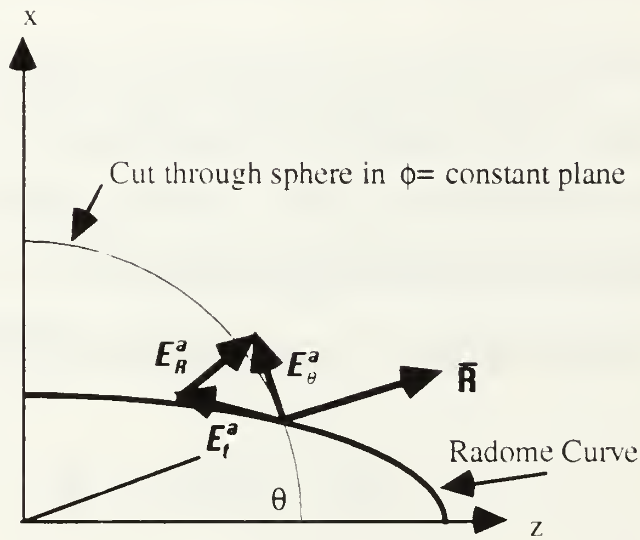

Figure 2.6 Components of the Antenna Electric-Field on the Radome

Plugging these far-field components into (2-19)

$$U_{ni}^t = \int_{t=0}^{t} \frac{T_i(t)}{\rho_s(t)} E_t^a(t)\rho(t)\,dt \int_{\phi=0}^{2\pi} \cos\phi\left[\cos(n\phi) + j\sin(n\phi)\right]d\phi$$
$$U_{ni}^\phi = \int_{t=0}^{t} \frac{P_i(t)}{\rho_s(t)} E_\phi^a(t)\rho(t)\,dt \int_{\phi=0}^{2\pi} \sin\phi\left[\cos(n\phi) + j\sin(n\phi)\right]d\phi. \qquad (2\text{-}37)$$

Only the ϕ integration is dependent on n. Expanding the integral in ϕ and explicitly including the sign of n gives

$$\int_{\phi=0}^{2\pi} \cos\phi\left[\cos(\pm n\phi) + j\sin(\pm n\phi)\right]d\phi =$$

$$\int_{\phi=0}^{2\pi} \cos\phi\cos(\pm n\phi)d\phi + j\int_{\phi=0}^{2\pi} \cos\phi\sin(\pm n\phi)d\phi, \tag{2-38}$$

where

$$\int_{\phi=0}^{2\pi} \cos\phi\sin(\pm n\phi)d\phi = 0, \tag{2-39}$$

due to the fact that the integrand is an odd function of ϕ, and the integration is over one period $0 \le \phi \le 2\pi$. By trigonometric identity

$$\int_{\phi=0}^{2\pi} \cos\phi\cos(\pm n\phi)d\phi = \int_{=0}^{2\pi} \cos\left[(\pm n - 1)\phi\right]d\phi + \int_{\phi=0}^{2\pi} \cos\left[(\pm n + 1)\phi\right]d\phi. \tag{2-40}$$

The right hand side is only non zero if $n = \pm 1$. Furthermore, because cosine is an even function, the term U^t_{ni} is independent of the sign of n. Thus

$$U^t_{-nj} = +U^t_{+nj}. \tag{2-41}$$

For the second equation of (2-37), the sign of U^o_{ni} is dependent on the integral

$$\int_{\phi=0}^{2\pi} \sin\phi\left[\cos(\pm n\phi) + j\sin(\pm n\phi)\right]d\phi =$$

$$\int_{\phi=0}^{2\pi} \sin\phi\cos(\pm n\phi)d\phi + j\int_{\phi=0}^{2\pi}\sin\phi\sin(\pm n\phi)d\phi, \tag{2-42}$$

where

$$\int_{\phi=0}^{2\pi} \sin\phi\cos(\pm n\phi)d\phi = 0, \tag{2-43}$$

by symmetry. By trigonometric identity

$$\int_{\phi=0}^{2\pi} \sin\phi\sin(\pm n\phi)d\phi = \int_{\phi=0}^{2\pi}\cos\left[(\pm n - 1)\phi\right]d\phi - \int_{\phi=0}^{2\pi}\cos\left[(\pm n + 1)\phi\right]d\phi. \tag{2-44}$$

As can be seen in (2-44), the excitation vector U°_{ni} is dependent on the sign of n due to the presence of the negative sign. Therefore

$$U^{\circ}_{-nj} = -U^{\circ}_{+nj}. \tag{2-45}$$

In the near-field of the antenna the ϕ dependence is more complicated than the sine and cosine functions in (2-36). However, it has been verified numerically that E^{a}_{θ} and E^{a}_{R} are always even functions of ϕ and E^{a}_{ϕ} is always odd. This conclusion assumes that a symmetric amplitude and linear phase distribution excites the antenna.

In light of the above results, the elements of the impedance matrix $\left[\mathbf{Z}_{MM}\right]$ of (2-18) have the following relationship between positive and negative modes: [Ref. 6]

$$[\mathbf{Z}_{\mathbf{MM}}] = \begin{bmatrix} \left[\mathbf{Z}^{tt}_{\mathbf{MM}(-n)}\right]_{ij} & \left[\mathbf{Z}^{t\phi}_{\mathbf{MM}(-n)}\right]_{ij} \\ \left[\mathbf{Z}^{\phi t}_{\mathbf{MM}(-n)}\right]_{ij} & \left[\mathbf{Z}^{\phi\phi}_{\mathbf{MM}(-n)}\right]_{ij} \end{bmatrix} = \begin{bmatrix} \left[\mathbf{Z}^{tt}_{\mathbf{MM}(+n)}\right]_{ij} & \left[-\mathbf{Z}^{t\phi}_{\mathbf{MM}(+n)}\right]_{ij} \\ \left[-\mathbf{Z}^{\phi t}_{\mathbf{MM}(+n)}\right]_{ij} & \left[\mathbf{Z}^{\phi\phi}_{\mathbf{MM}(+n)}\right]_{ij} \end{bmatrix}. \quad (2\text{-}46)$$

By inspection of (2-30), the elements of the load impedance matrix $[\mathbf{Z}_{\mathbf{L}}]$ are independent of the mode n, because $\tilde{\boldsymbol{W}}^{t\phi}_{in}$ and $\tilde{\mathbf{J}}^{t\phi}_{jn}$ are complex conjugates of each other. Therefore,

$$[\mathbf{Z}_{\mathbf{L}}] = \begin{bmatrix} \left[\mathbf{Z}^{tt}_{\mathbf{L}(-n)}\right]_{ij} & [\boldsymbol{\theta}] \\ [\boldsymbol{\theta}] & \left[\mathbf{Z}^{\phi\phi}_{\mathbf{L}(-n)}\right]_{ij} \end{bmatrix} = \begin{bmatrix} \left[\mathbf{Z}^{tt}_{\mathbf{L}(+n)}\right]_{ij} & [\boldsymbol{\theta}] \\ [\boldsymbol{\theta}] & \left[\mathbf{Z}^{\phi\phi}_{\mathbf{L}(+n)}\right]_{ij} \end{bmatrix}. \quad (2\text{-}47)$$

Rewriting (2-31) results in

$$[\mathbf{I}] = [\mathbf{Y}][\mathbf{U}], \quad (2\text{-}48)$$

where

$$[\mathbf{Y}] = [\mathbf{Z}_{\mathbf{MM}} + \mathbf{Z}_{\mathbf{L}}]^{-1}. \quad (2\text{-}49)$$

By taking the inverse through partitioning [Ref. 8] it can be shown that the mode symmetry survives the inversion, and

$$\begin{bmatrix} \left[\mathbf{Y}^{tt}_{-nij}\right] & \left[\mathbf{Y}^{t\phi}_{-nij}\right] \\ \left[\mathbf{Y}^{\phi t}_{-nij}\right] & \left[\mathbf{Y}^{\phi\phi}_{-nij}\right] \end{bmatrix} = \begin{bmatrix} \left[\mathbf{Y}^{tt}_{+nij}\right] & \left[-\mathbf{Y}^{t\phi}_{+nij}\right] \\ \left[-\mathbf{Y}^{\phi t}_{+nij}\right] & \left[\mathbf{Y}^{\phi\phi}_{+nij}\right] \end{bmatrix}. \quad (2\text{-}50)$$

By multiplying out (2-48),

$$\mathbf{I}^{t}_{n} = \mathbf{Y}^{tt}_{nij} U^{t}_{nj} + \mathbf{Y}^{t\phi}_{nij} U^{\phi}_{nj}$$
$$\mathbf{I}^{\phi}_{n} = \mathbf{Y}^{\phi t}_{nij} U^{t}_{nj} + \mathbf{Y}^{\phi\phi}_{nij} U^{\phi}_{nj}. \tag{2-51}$$

Equations (2-41), (2-45), and (2-50) relate the positive and negative modes of (2-51), as follows:

$$\mathbf{I}^{t}_{-n} = \mathbf{Y}^{tt}_{-nij} U^{t}_{+nj} - \mathbf{Y}^{t\phi}_{+nij}\left(-U^{\phi}_{+nj}\right) = +\mathbf{I}^{t}_{+n}$$
$$\mathbf{I}^{\phi}_{-n} = -\mathbf{Y}^{\phi t}_{-nij} U^{t}_{+nj} + \mathbf{Y}^{\phi\phi}_{+nij}\left(-U^{\phi}_{+nj}\right) = -\mathbf{I}^{\phi}_{+n}. \tag{2-52}$$

This is an extremely important result. Using mode symmetry reduces the number of calculations required to find the current coefficients $\mathbf{I}^{t,\phi}_{n}$ by nearly half in most cases.

3. Measurement Matrices for the Scattered Field

Once the current $\bar{\mathbf{J}}_s$ is obtained the scattered field $\bar{\mathbf{E}}^s$ can be determined by summing the contributions of the individual current elements in the far-field as shown in Figure 2.7. This is accomplished by use of a measurement vector with elements

$$\mathbf{R}_m = \iint_s \left(\bar{\mathbf{R}} + \bar{\theta} + \bar{\phi}\right) e^{-jkr} \bullet \bar{\mathbf{J}}_s ds. \qquad (2\text{-}53)$$

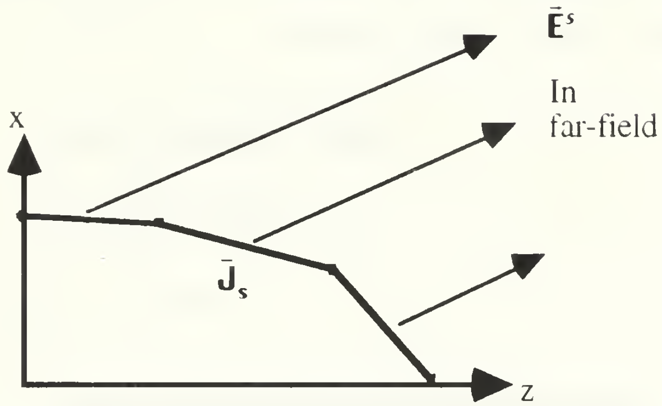

Figure 2.7 Scattered Field

In (2-53), a unit radiated plane wave is weighted by the current element at each point on the surface. Neglecting the $\bar{\mathbf{R}}$ component in the far-field, the measurement vector separated into θ and ϕ components is

$$R_m^\theta = \iint_s e^{-jk(xu+yv+wz)} \left(\bar{\theta} \bullet \bar{\mathbf{J}}_s\right) ds'$$

$$R_m^{\phi} = \iint_s e^{-jk(xu+yv+wz)}\left(\bar{\phi} \bullet \mathbf{J}_s\right)ds', \tag{2-54}$$

where u, v, and w are the direction cosines,

$$\begin{aligned} u &= \sin\theta\cos\phi \\ v &= \sin\theta\sin\phi \\ w &= \cos\theta. \end{aligned} \tag{2-55}$$

Thus, the components of the scattered field at a far-field point are the superposition of all surface segments

$$\begin{aligned} E_\theta^s &= \frac{-jk\eta}{4\pi r} e^{-jkr} \sum_m R_m^\theta I_m \\ E_\phi^s &= \frac{-jk\eta}{4\pi r} e^{-jkr} \sum_m R_m^\phi I_m. \end{aligned} \tag{2-56}$$

4. Closed-Form Far-Field Antenna Pattern

When computing the antenna contribution to the total field in the far zone the higher order $\frac{1}{R}$ terms can be neglected, and the radiation integral can be reduced to a closed form. The source of this field is assumed to be a uniformly-excited circular-aperture antenna scanned to a direction $\left(\theta_s, \phi_s\right)$. Uniformly excited refers to a constant amplitude and a linear phase distribution of current on the antenna. The R, θ and ϕ components in the far-field are

$$E_R^a \cong 0$$

$$E_\theta^a \cong \frac{-jk}{4\pi}\frac{e^{-jkr}}{r}\left(L_\phi + \eta N_\theta\right)$$

$$E_\phi^a \cong \frac{-jk}{4\pi}\frac{e^{-jkr}}{r}\left(L_\theta + \eta N_\phi\right), \tag{2-57}$$

where L_ϕ, L_θ N_θ and N_ϕ are given by

$$\bar{\mathbf{L}} = \iint_{s'} \bar{\mathbf{M}} e^{-jkr'\cos\psi'} ds' = \bar{\mathbf{0}}, \ (\bar{\mathbf{M}} = \bar{\mathbf{0}}),$$

$$\bar{\mathbf{N}} = \iint_{s'} \bar{\mathbf{J}} e^{-jkr'\cos\psi'} ds' = \iint_{s'} \left(\bar{\mathbf{x}}J_x + \bar{\mathbf{y}}J_y + \bar{\mathbf{z}}J_z\right) e^{-jkr'\cos\psi'} ds' \tag{2-58}$$

where r' and ψ' are as shown in Figure 2.8 [Ref. 10:pp. 455]. If the antenna is x-polarized, then $N_y = N_z = 0$ and

$$N_x = \iint_{s'} J_x e^{-jkr'\cos\psi'} ds'. \tag{2-59}$$

The equivalent in spherical coordinates is

$$N_\theta = \iint_{s} J_x \cos\theta \cos\phi \, e^{-jkr'\cos\psi'} ds'$$

$$N_\phi = \iint_{s} J_x \sin\phi \, e^{jkr'\cos\psi'} ds'. \tag{2-60}$$

The phase of the source point relative to the field point, $r' \cos \psi'$, is a length that can be written in Cartesian coordinates as

$$r' \cos \psi' = x'u + y'v + z'w, \qquad (2\text{-}61)$$

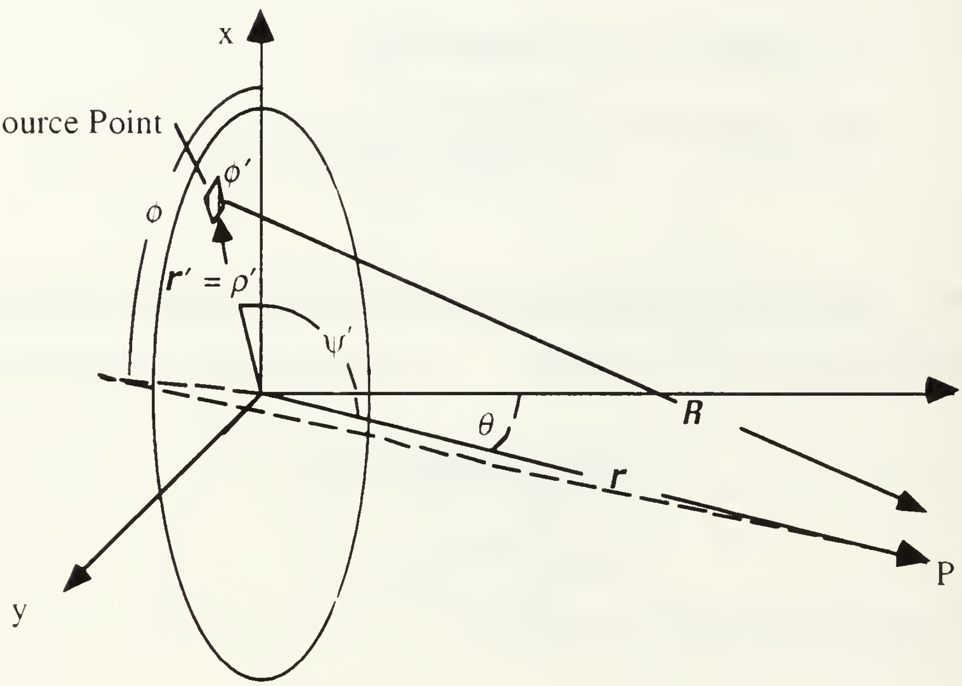

Figure 2.8 Circular Aperture Antenna Source Point

where u, v, and w are the direction cosines (2-55). Including beam scanning adds an exponential phase factor to the current

$$J_x = J_o e^{jk(x'u_s + y'v_s + z'w_s)}, \qquad (2\text{-}62)$$

where u_s, v_s and w_s are the direction cosines of the scan angles (θ_s, ϕ_s) which also obey (2-55). J_o is the magnitude distribution, assumed to be constant, and (x', y', z') are the coordinates of a source point. For an antenna lying in the x-y plane, $z' = 0$. Therefore, N_θ can be rewritten as

$$
\begin{aligned}
N_\theta &= J_o \cos\theta \cos\phi \iint_{s'} e^{jk(x'u_s + y'v_s)} e^{-jk(x'u + y'v)} ds'. \\
&= J_o \cos\theta \cos\phi \iint_{s'} e^{jk(x'(u_s - u) + y'(v_s - v))} ds'.
\end{aligned} \tag{2-63}
$$

By setting $x' = \rho' \cos\phi'$ and $y' = \rho' \sin\phi'$, the exponent term of (2-63) becomes

$$
k\rho'\left[\cos\phi'(u_s - u) + \sin\phi'(v_s - v)\right]. \tag{2-64}
$$

Defining the variables

$$
\begin{aligned}
A &= (u_s - u) \\
B &= (v_s - v),
\end{aligned} \tag{2-65}
$$

and considering these to be orthogonal vectors as in Figure 2.9, it is possible to rewrite (2-64) as

$$
k\rho'\sqrt{A^2 + B^2}\left[\cos\phi' \frac{A}{\sqrt{A^2 + B^2}} + \sin\phi' \frac{B}{\sqrt{A^2 + B^2}}\right]. \tag{2-66}
$$

From Figure 2.9 the factors containing A and B in the brackets can be replaced with $\cos\gamma$ and $\sin\gamma$

$$k\rho'\sqrt{A^2+B^2}\left[\cos\phi'\cos\gamma+\sin\phi'\sin\gamma\right]. \tag{2-67}$$

By trigonometric identity,

$$k\rho'\sqrt{A^2+B^2}\left[\cos\left(\phi'-\gamma\right)\right]. \tag{2-68}$$

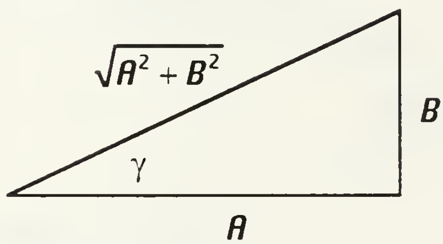

Figure 2.9 Representation of A and B as vectors

Using this form of the exponent in the integral of (2-63), gives

$$N_\theta = J_o\cos\theta\cos\phi\int_{\phi'=0}^{2\pi}\int_{\rho'=0}^{a} e^{jk\rho'\sqrt{A^2+B^2}\cos\left(\phi'-\gamma\right)}\rho' d\rho' d\phi', \tag{2-69}$$

which conforms to the definition of a Bessel function. The same integration is performed in the second equation of (2-60), and the same Bessel function results. Finally,

$$N_\theta = J_o \cos\theta \cos\phi \; \pi a^2 \left[2\frac{J_1\left(ka\sqrt{A^2+B^2}\right)}{ka\sqrt{A^2+B^2}} \right]$$

$$N_\phi = -J_o \sin\phi \; \pi a^2 \left[2\frac{J_1\left(ka\sqrt{A^2+B^2}\right)}{ka\sqrt{A^2+B^2}} \right], \qquad (2\text{-}70)$$

where J_1 is a Bessel function of the first order. The far-field equations (2-57) become

$$E_\theta = -J_o \frac{jk\eta}{4\pi} \frac{e^{-jkr}}{r} \cos\theta \cos\phi \; \pi a^2 \left[\frac{2J_1\left(ka\sqrt{A^2+B^2}\right)}{ka\sqrt{A^2+B^2}} \right]$$

$$E_\phi = J_o \frac{jk\eta}{4\pi} \frac{e^{-jkr}}{r} \sin\phi \; \pi a^2 \left[\frac{2J_1\left(ka\sqrt{A^2+B^2}\right)}{ka\sqrt{A^2+B^2}} \right], \qquad (2\text{-}71)$$

where

$$A = u_s - u = \sin\theta_s \cos\phi_s - \sin\theta \cos\phi$$

$$B = v_s - v = \sin\theta_s \sin\phi_s - \sin\theta \sin\phi, \qquad (2\text{-}72)$$

and

$$\sqrt{A^2 + B^2} = \sqrt{\sin^2\theta + \sin^2\theta_s - 2\sin\theta_s \sin\theta\cos(\phi - \phi_s)}. \qquad (2\text{-}73)$$

Using equations (2-71) through (2-73) are computationally less intensive than a numerical integration. These equations were employed for both the far field pattern and gain computations.

5. **Near Field of the Antenna**

In the previous section, the far-field of a circular aperture antenna was determined. When a radome is located in the near-field, the integrals cannot be reduced to closed form and they must be evaluated numerically. The near field is generally considered to be the region surrounding the antenna where $r << \lambda$, or $kr << 1$ [Ref. 10:pp. 106].

The complete expressions for the near-field of a circular aperture antenna excited by a current polarized in the x-direction are

$$\begin{aligned} E_x &= \frac{-j\eta}{4\pi k}\iint_{s'}\left[G_1 J_x + (x - x')^2 G_2 J_x\right]e^{-jkr}\,dx'dy' \\ E_y &= \frac{-j\eta}{4\pi k}\iint_{s'}\left[(x - x')(y - y')G_2 J_x\right]e^{-jkr}\,dx'dy' \\ E_z &= \frac{-j\eta}{4\pi k}\iint_{s'}\left[(x - x')(z - z')G_2 J_x\right]e^{-jkr}\,dx'dy', \end{aligned} \qquad (2\text{-}74)$$

where

$$\begin{aligned} G_1 &= \frac{k^2 r^2 - 1 - jkr}{r^3} \\ G_2 &= \frac{3 + 3jkr - k^2 r^2}{r^5}, \end{aligned} \qquad (2\text{-}75)$$

and

$$r = \sqrt{\left(x - x'\right)^2 + \left(y - y'\right)^2 + \left(z - z'\right)^2}\,. \tag{2-76}$$

The primed coordinates designate source points and the unprimed an observation point on the radome as in Figure 2.10. [Ref. 11] S' is the area of the antenna aperture. Since the radome is rotationally symmetric, the components $\left(E_x, E_y, E_z\right)$ can be described in terms of cylindrical coordinates using the transformations

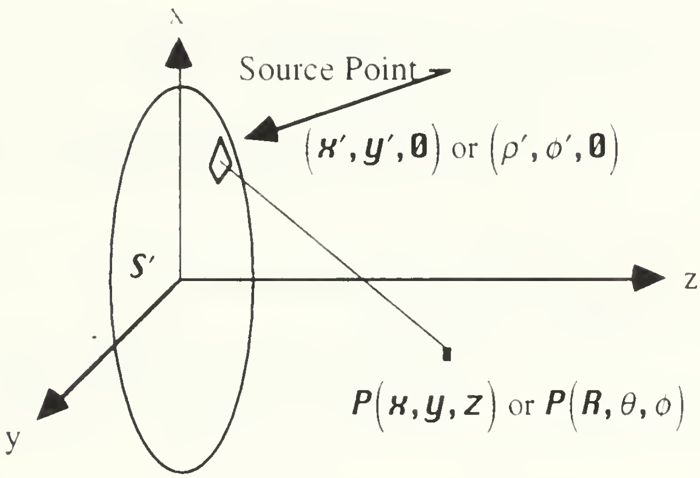

Figure 2.10 Rectangular Coordinates for Circular Aperture Antenna

$$\begin{aligned} x' &= \rho' \cos\phi' \\ y' &= \rho' \sin\phi' \\ dx'dy' &= \rho' d\rho' d\phi'. \end{aligned} \tag{2-77}$$

For an antenna scanned to $\left(\theta_s, \phi_s\right)$ the current distribution is

$$J_x = J_o(\rho')e^{jk\left(x'\cos\left(\phi'-\phi_s\right)\sin\theta_s + y'\sin\left(\phi'-\phi_s\right)\sin\theta_s\right)}. \quad (2\text{-}78)$$

Applying (2-75), 2-76) and (2-77) to equations (2-74) results in

$$E_x = \frac{-j\eta}{4\pi k}\int_{\phi'=0}^{2\pi}\int_{\rho'=0}^{a} J_o(\rho')e^{jk\rho'\cos(\phi-\phi_s)\sin\theta_s}e^{-jkr}\times \left[G_1 + \left(\rho\cos\phi-\rho'\cos\phi'\right)^2 G_2\right]\rho' d\rho' d\phi'$$

$$E_y = \frac{-j\eta}{4\pi k}\int_{\phi'=0}^{2\pi}\int_{\rho'=0}^{a} J_o(\rho')e^{jk\rho'\cos(\phi-\phi_s)\sin\theta_s}e^{-jkr}\times \left[\left(\rho\cos\phi-\rho'\cos\phi'\right)\left(\rho\sin\phi-\rho'\sin\phi'\right)G_2\right]\rho' d\rho' d\phi'$$

$$E_z = \frac{-j\eta}{4\pi k}\int_{\phi'=0}^{2\pi}\int_{\rho'=0}^{a} J_o(\rho')e^{jk\rho'\cos(\phi-\phi_s)\sin\theta_s}e^{-jkr}\times \left[\left(\rho\cos\phi-\rho'\cos\phi'\right)zG_2\right]\rho' d\rho' d\phi'. \quad (2\text{-}79)$$

where

$$r = \sqrt{\left(\rho-\rho'\right)^2 + \left(z-z'\right)^2 + 4\rho\rho'\sin^2\left(\frac{\phi-\phi'}{2}\right)}. \quad (2\text{-}80)$$

Since the expression for the excitation vector requires spherical components, $\left(E_x, E_y, E_z\right)$ are converted using a coordinate transformation

$$E_R = E_x \sin\theta\cos\phi + E_y \sin\theta\sin\phi + E_z \cos\theta$$
$$E_\theta = E_x \cos\theta\cos\phi + E_y \cos\theta\sin\phi - E_z \sin\theta$$
$$E_\phi = -E_x \sin\phi + E_y \cos\phi. \tag{2-81}$$

These are the equations programmed to compute the E-field impinging on the radome. The magnitude of the current distribution is allowed to vary as a function of ρ'. Various antenna sidelobe levels can be modeled by simply changing the function $J_o(\rho')$

III. PROGRAMS

The mathematical formulas developed in the previous chapters were computer programmed using FORTRAN. The pattern and gain calculations are done in two separate programs: *RADOME.F* and *GAIN.F*, both of which run on a Sun SPARCstation™ under UNIX. A brief description of each follows.

A. DESCRIPTION OF THE PROGRAM RADOME.F

This program, developed by Francis [Ref. 2], determines the series expansion coefficients of the current $\bar{\mathbf{J}}_s$ induced on the surface of the radome. It also calculates the total electric field surrounding the system, and generates radiation patterns using MATLAB™. Originally called *LDBOR.F*, (an abbreviation of "Loaded Bodies of Revolution") in Ref. 2, it has been modified to take advantage of symmetry, and also allows the antenna mainbeam to scan in both θ and ϕ.

Figure 3.1 is a flow chart of the program. To run the program, data is entered interactively, except for a data file "gaus/gaus(ITH)" containing the integration constants for Gauss Quadrature, which is read in the program. All integrals are evaluated using this technique (see Appendix A.). Table I is a sample input.

After all data is entered, the coordinate points of the surface are computed and stored in the arrays RH($=\rho$) and ZH($=\boldsymbol{z}$). The surface impedance of each segment is stored in ZL0. The elements of the MM impedance matrix are computed in subroutine ZMAT, and the excitation vector elements in GENEX. These provide $\left[\mathbf{Z}_{MM}\right]$ and $\left[\mathbf{U}\right]$ in equation (2-17). Subroutines DECOMP and SOLVE solve the matrix equation using Gaussian elimination. Subroutine PLANE calculates the measurement matrices, which are subsequently used to compute the scattered field from the radome.

Two antenna radiation subroutines are included. CIRCRTP calculates the electric field components E_θ, E_ϕ, and E_ρ according to equations (2-81). It is used by subroutine GENEX to obtain the excitation vector. The second antenna subroutine ANTFF assumes that the observation point is in the far-field. This is used in the main program to compute the antenna contribution to the radiation pattern $\vec{E}^a$ in (2-3).

Output includes an ASCII file "outldbor" which lists all the input parameters, radome geometry and impedance, and the pattern magnitude. The current coefficients are written on a file "curcoefsdat" so that further patterns can be generated without recomputing the coefficients. Finally, MATLAB™ formatted files are generated for plotting the θ and ϕ components of the electric field.

TABLE I. SAMPLE INPUT RADOME.F

ENTER A LETTER TO INDICATE WHICH ITERATION THIS IS	A
SELECT BOR GEOMETRY BY NUMBER	
NUMBER BOR	
1 OGIVE	1
2 SPHERE	
3 CONE	
4 DISK	
5 PARABOLA	
ENTER SURFACE CURVATURE (wavelengths)	7.5
ENTER ZPRIME,WHERE CURVATURE STARTS (wavelengths)	0
ENTER BASE RADIUS (wavelengths)	3
ENTER FILENAMES gaus###	
ENTER THE FILENAME IN T (NT)	gaus2
ENTER THE FILENAME IN PHI (NPHI)	gaus48
ENTER THE FILENAME IN X AND Y	gaus20
ENTER THE PLOTTING INCREMENT IN DEGREES	1
ENTER THE HIGHEST MODE	20
ENTER PHI (observation) IN DEGREES	90
ENTER SCAN ANGLES IN DEGREES: THETA, PHI	30,90
ENTER COMPLEX IMPEDANCE, OHMS: (Real, Imag)	(0,-1700)
ENTER THE ANTENNA RADIUS (wavelengths)	1

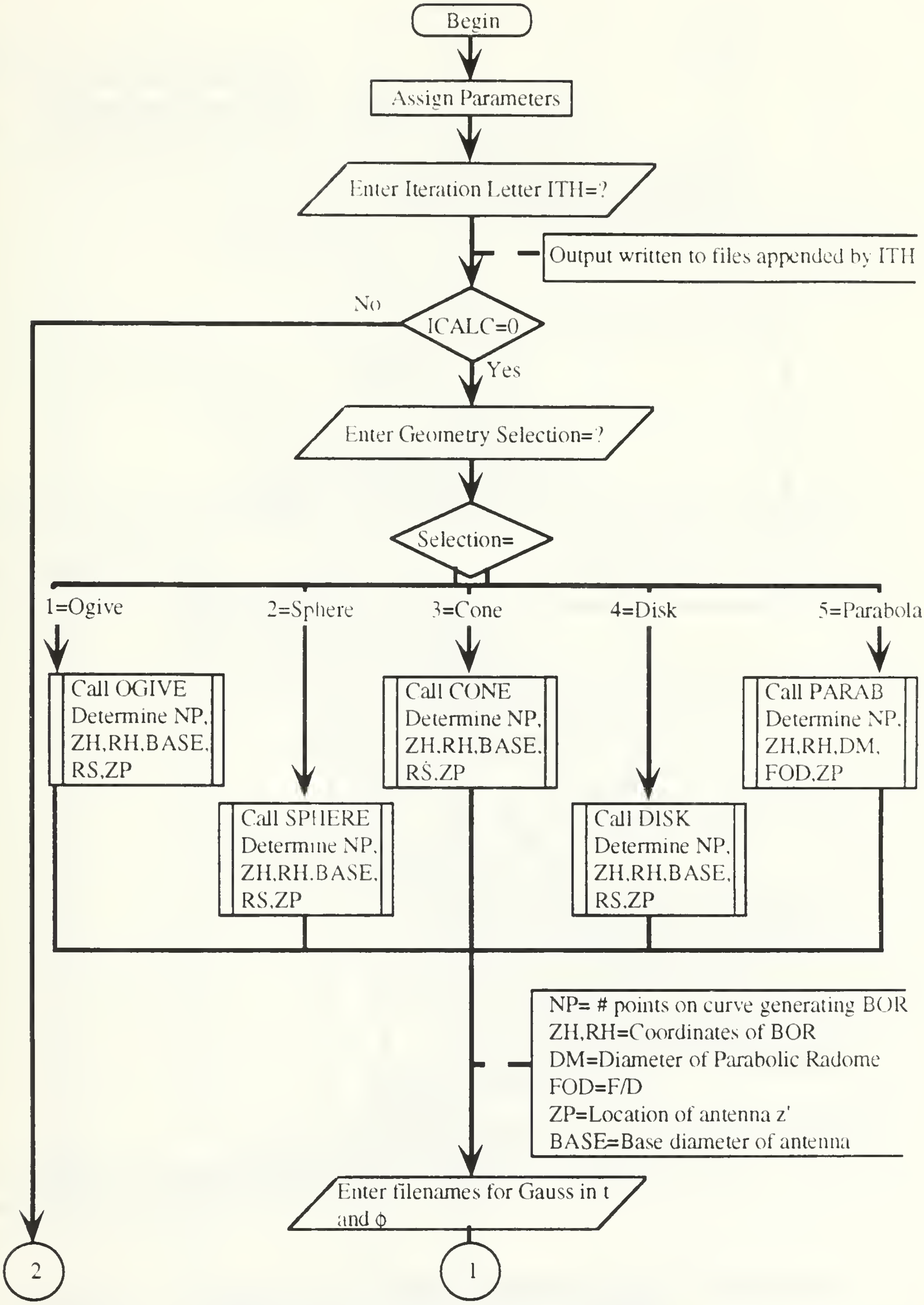

Figure 3.1 *RADOME.F* **Flow Diagram**

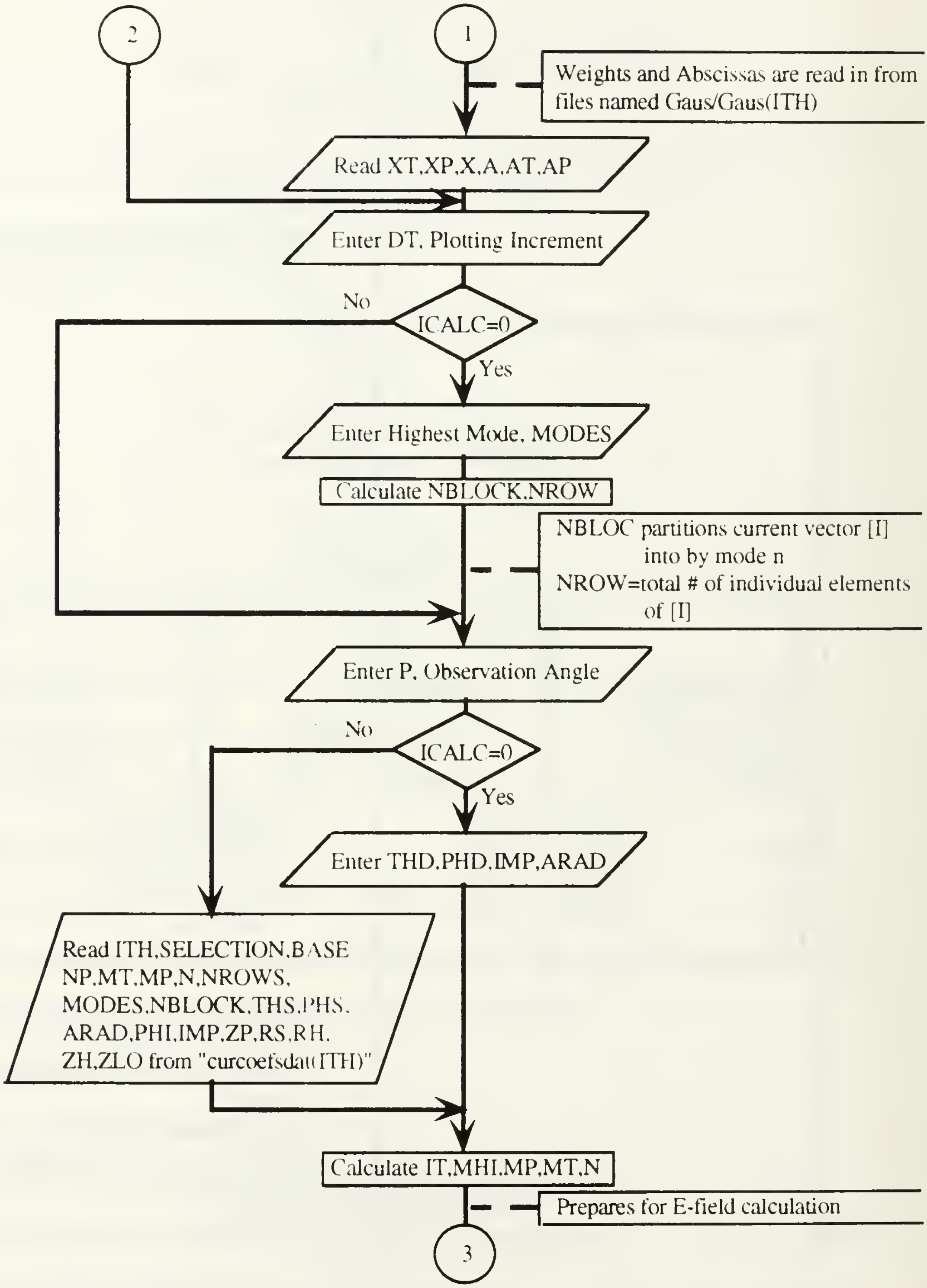

Figure 3.1 *RADOME.F* **Flow Diagram (Continued)**

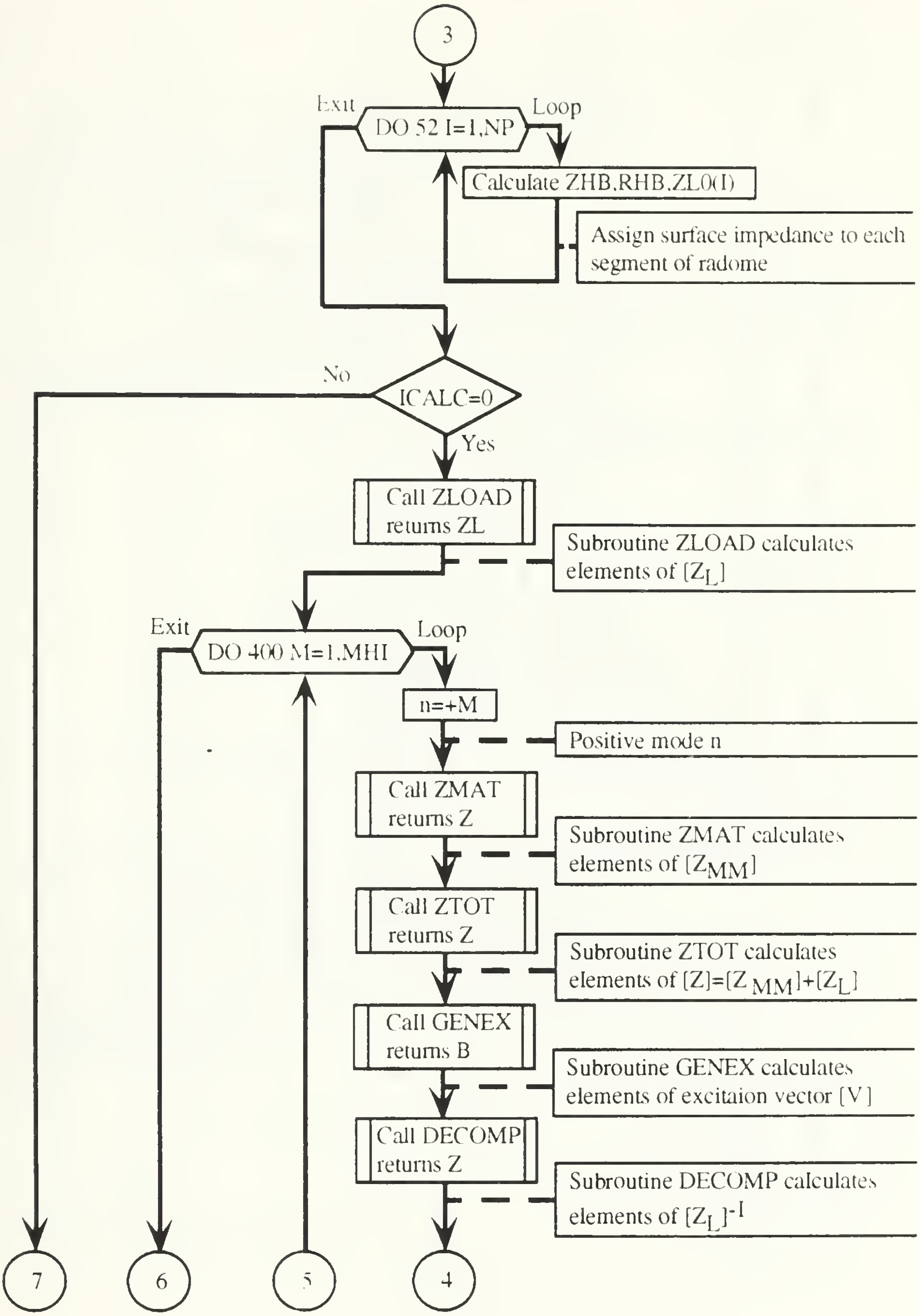

Figure 3.1 *RADOME.F* **Flow Diagram (Continued)**

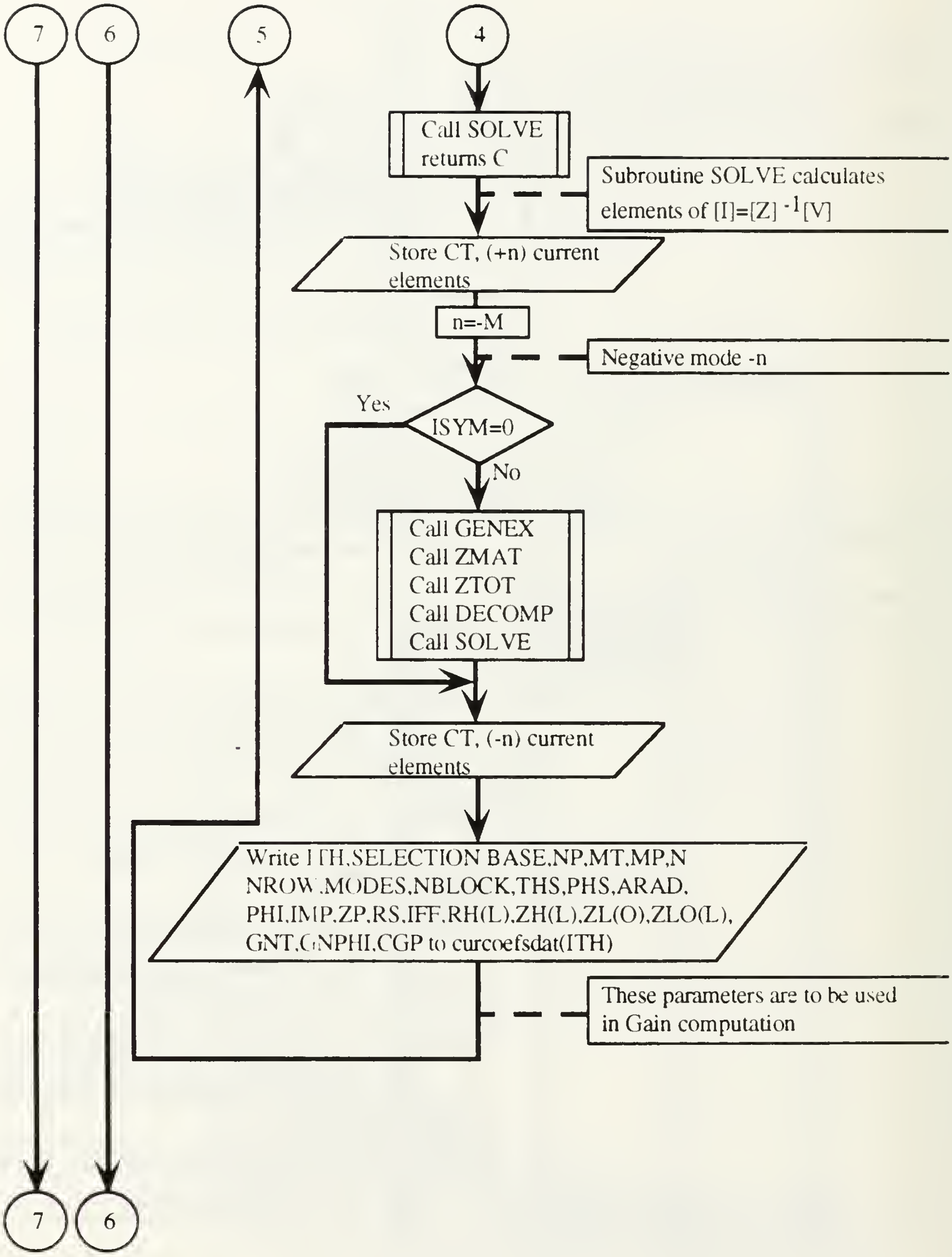

Figure 3.1 ***RADOME.F*** **Flow Diagram (Continued)**

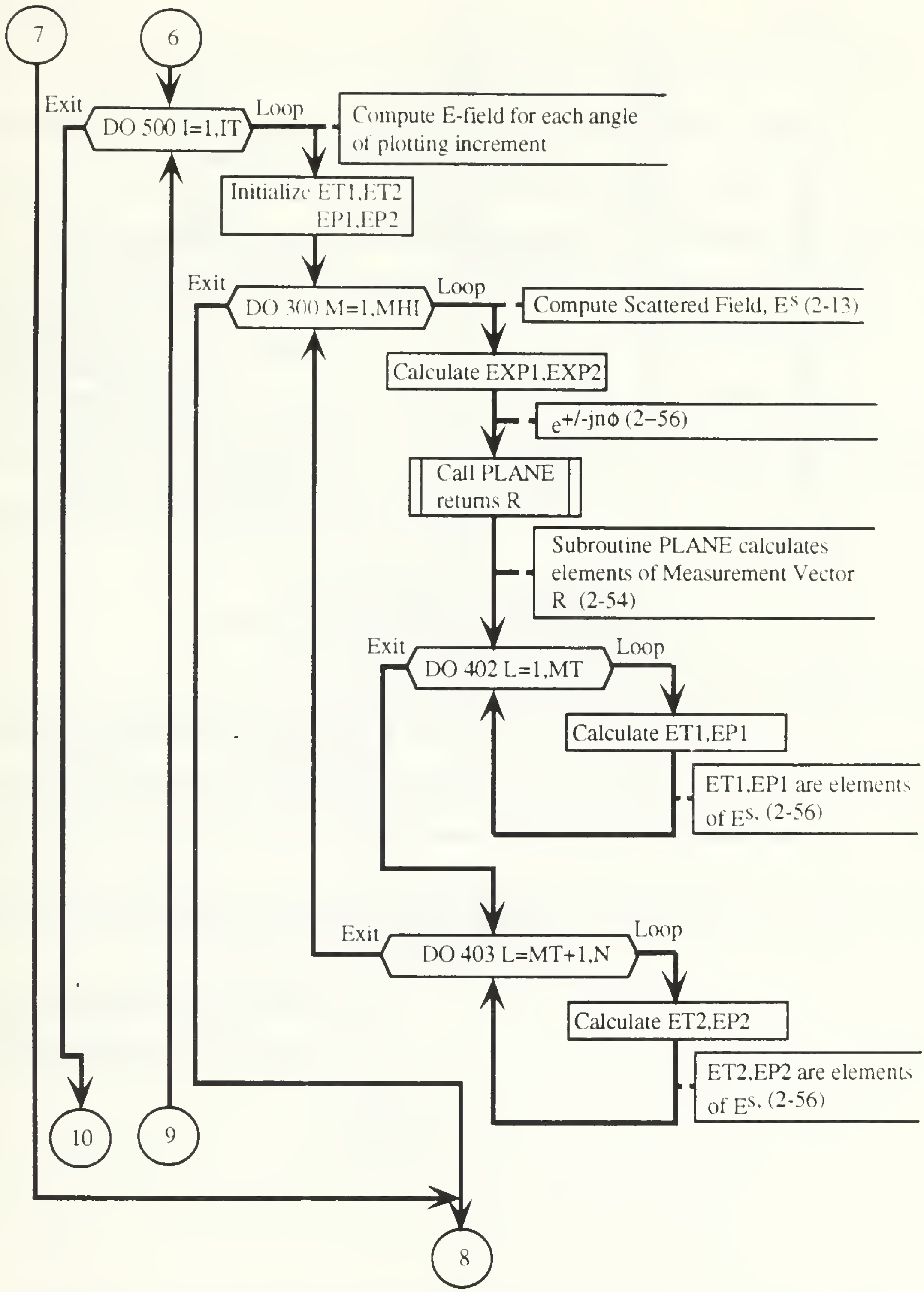

Figure 3.1 ***RADOME.F*** **Flow Diagram (Continued)**

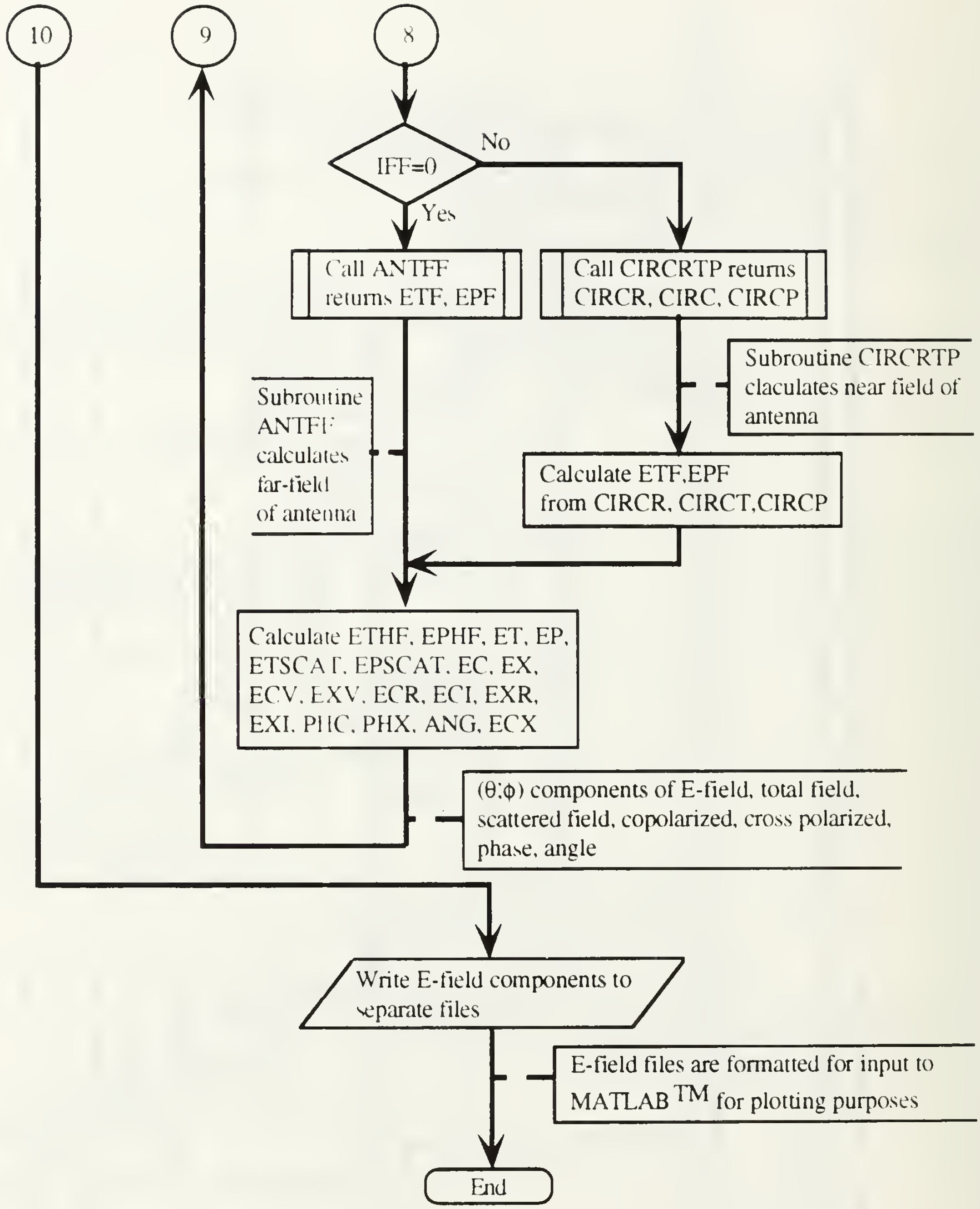

Figure 3.1 *RADOME.F* **Flow Diagram (Continued)**

B. GAIN.F

This program determines the gain of the system by solving (2-1). In order to perform the integration, the current coefficients from *RADOME.F* must be provided so that the radome scattered field can be computed. Input consists of specifying an iteration letter, and selecting the number of divisions in the θ and ϕ integration. Input data is then read from a file "curcoefsdat" that is appended with the iteration letter chosen.

Figure 3.2 is a flow chart of the program, and Table II is a sample input. *GAIN.F* contains several of the same subroutines that occur in *RADOME.F*. They include PLANE, CIRCRTP, and ANTFF.

TABLE II. SAMPLE INPUT GAIN.F

ENTER A LETTER TO INDICATE WHICH ITERATION THIS IS	A
ENTER NUMBER OF DIVISIONS IN THETA DIRECTION,	
NDIVT = ?	2
ENTER NUMBER OF DIVISIONS IN PHI DIRECTION, NDIVP =?	3

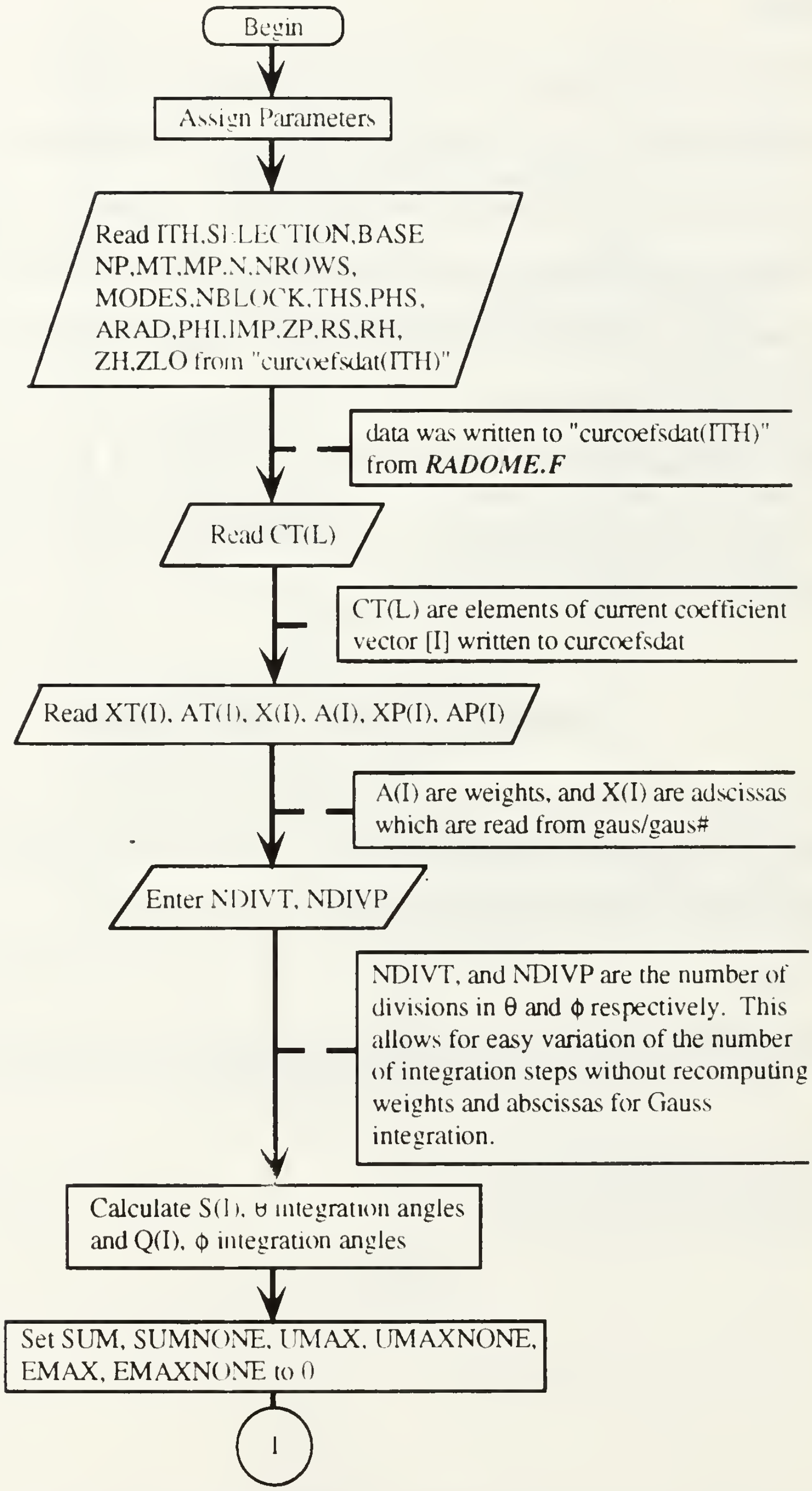

Figure 3.2 ***GAIN.F*** **Flow Diagram**

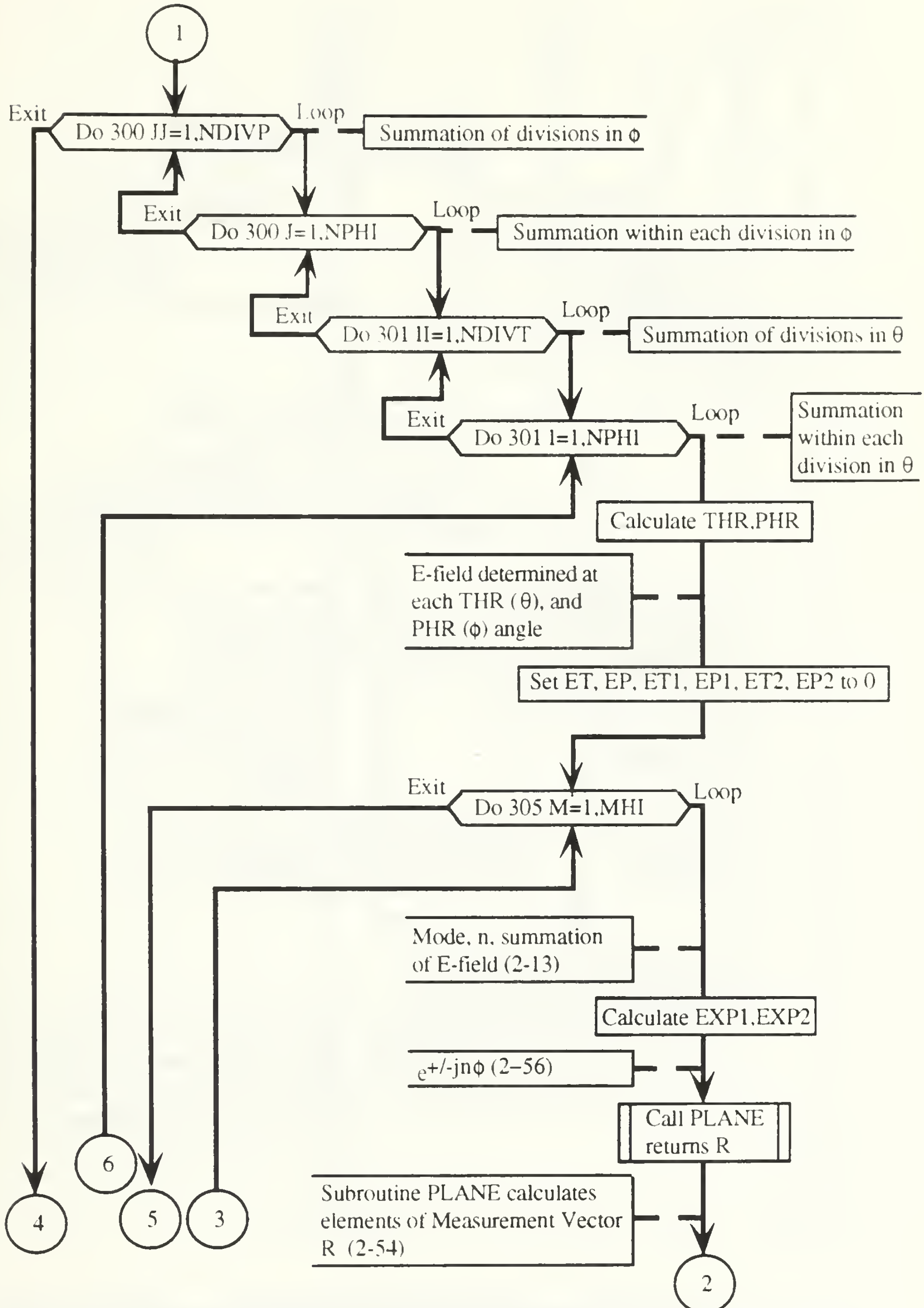

Figure 3.2 ***GAIN.F*** **Flow Diagram (Continued)**

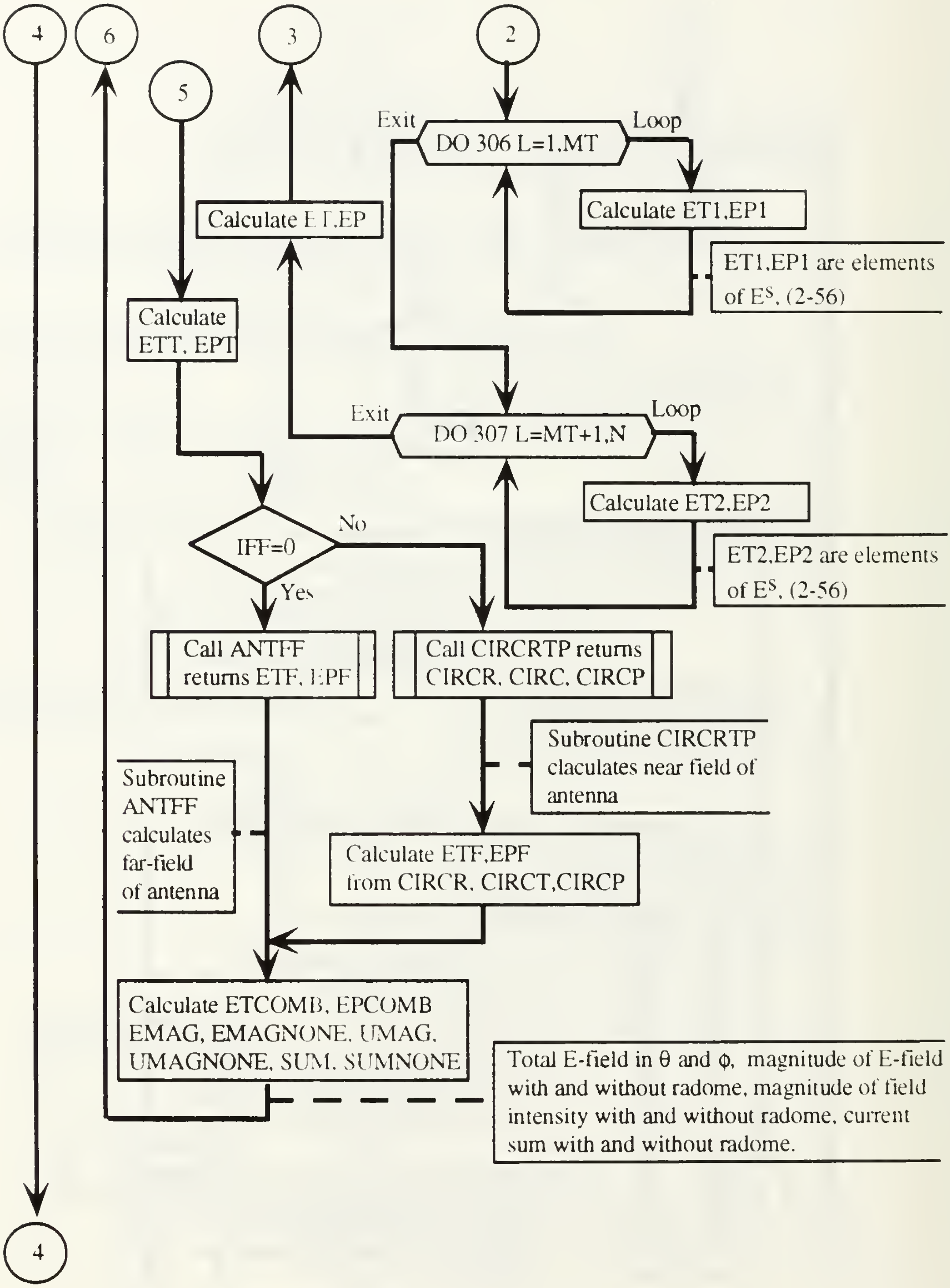

Figure 3.2 ***GAIN.F*** **Flow Diagram (Continued)**

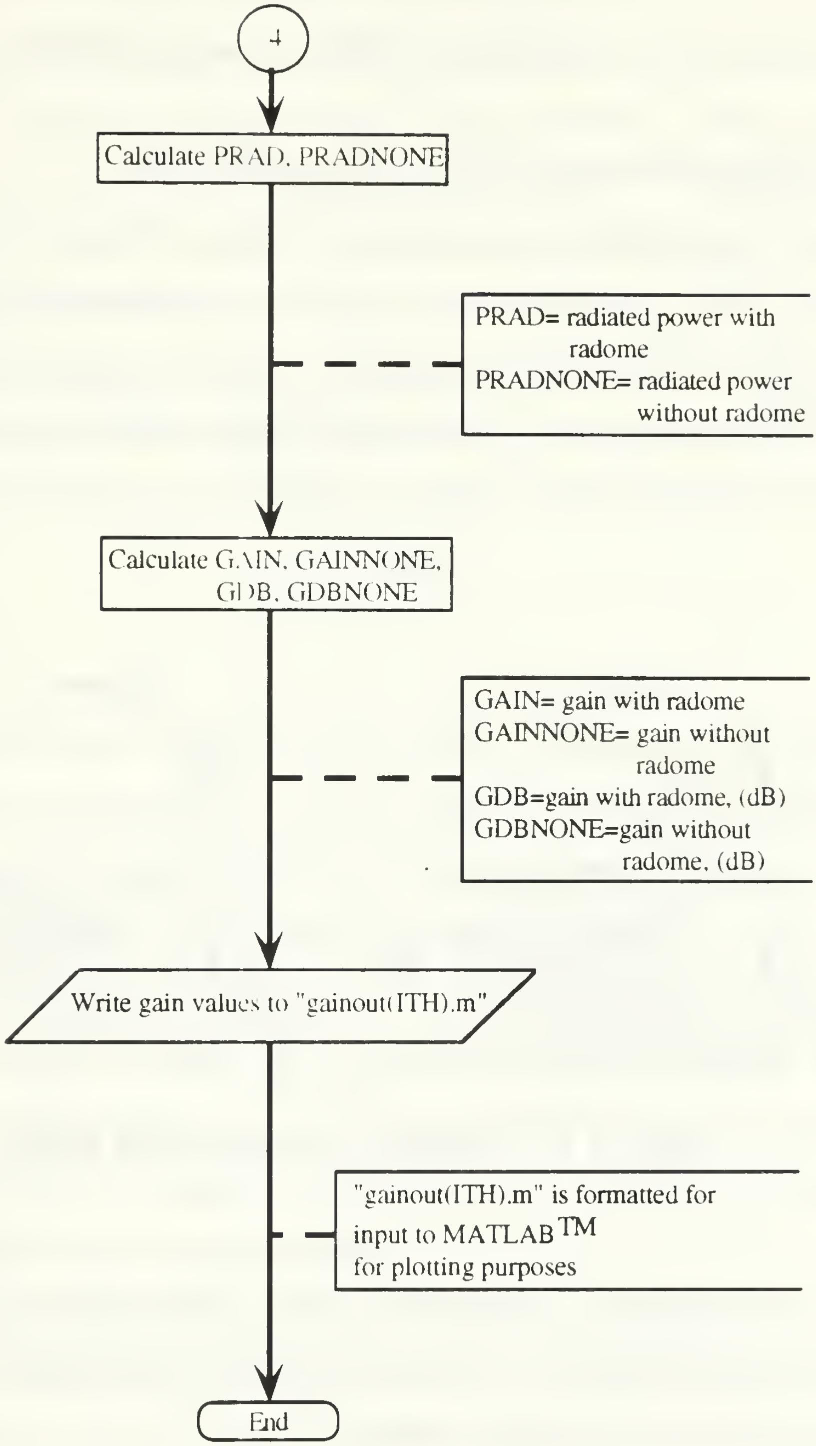

Figure 3.2 *GAIN.F* **Flow Diagram (Continued)**

IV. RESULTS AND CONCLUSIONS

A. VALIDATION AND CALCULATED RESULTS

Part of the continued development of this research is validation of the programs. Test cases were run to see how the programs performed against known results. Test radomes ranged from nearly nonexistent disks located at large distances, to perfectly conducting parabolas located in the near-field. E-plane and H-plane patterns defined in Figure 4.1 were plotted to determine the

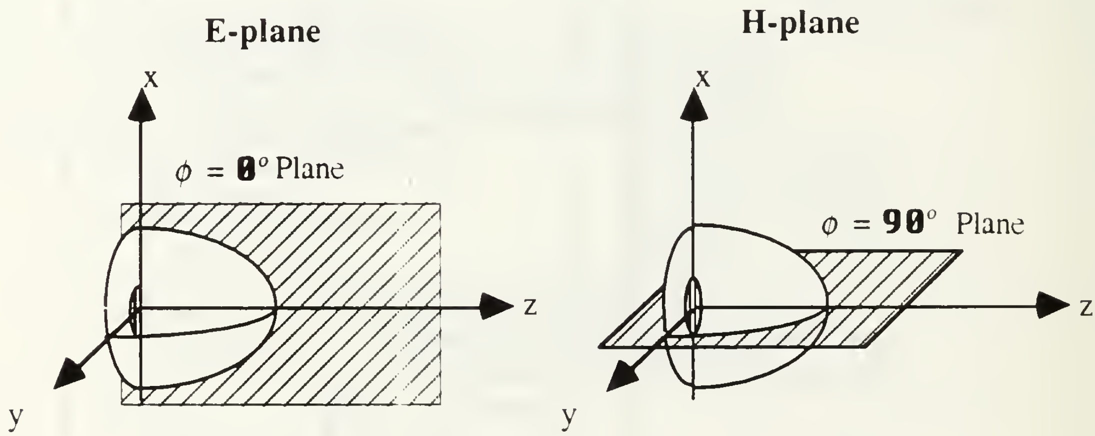

Figure 4.1 Orientation of E-plane and H-plane

effects of the radome. The antenna pattern without the radome was plotted (dashed line) along with the pattern of the complete system (solid line) to illustrate the effects of radome scattering.

1. Code Validation: Scattering from a Small Disk

The pattern of an antenna radiating in the presence of a negligibly-small circular-disk located at a large distance was computed. Since the radome scattered field is negligible, the predicted output is that of a uniformly-excited circular-aperture antenna. Figure 4.2 shows the plots of the E-plane pattern for three different radius antennas (.5 λ, 1.0 λ and 2.0 λ). The plots appear as single lines with no scattering. The gain as computed by *GAIN.F*, was compared to the theoretical gain of a circular aperture with a uniform current distribution

$$G_o = \left(\frac{2\pi a}{\lambda}\right)^2 . \tag{4-1}$$

where a is the aperture radius [Ref. 10:pp. 483]. Theoretical gains for the .5 λ, 1.0 λ and 2.0 λ radius apertures are 9.94, 15.96, and 21.98 dB respectively. In each case the numerical gain is within 1.5 dB of the theoretical gain.

2. Code Validation: Perfectly Conducting Paraboloid

Programs have been written that use the MM solution for bodies of revolution developed by Mautz and Harrington to predict the patterns and gains of reflector antennas [Ref. 7]. Assuming that the radome is a perfectly conducting parabola, the output of *RADOME.F* and *GAIN.F* can be compared with the output of these existing programs.

Figure 4.3 is a test case where the radome is a parabola with surface impedance $Z_l = 0 + j0\ \Omega$. The dimensions of the radome matched those of a reflector which was analyzed by a separate computer program that calculates both pattern, and gain. The pattern and gain for the perfectly conducting radome

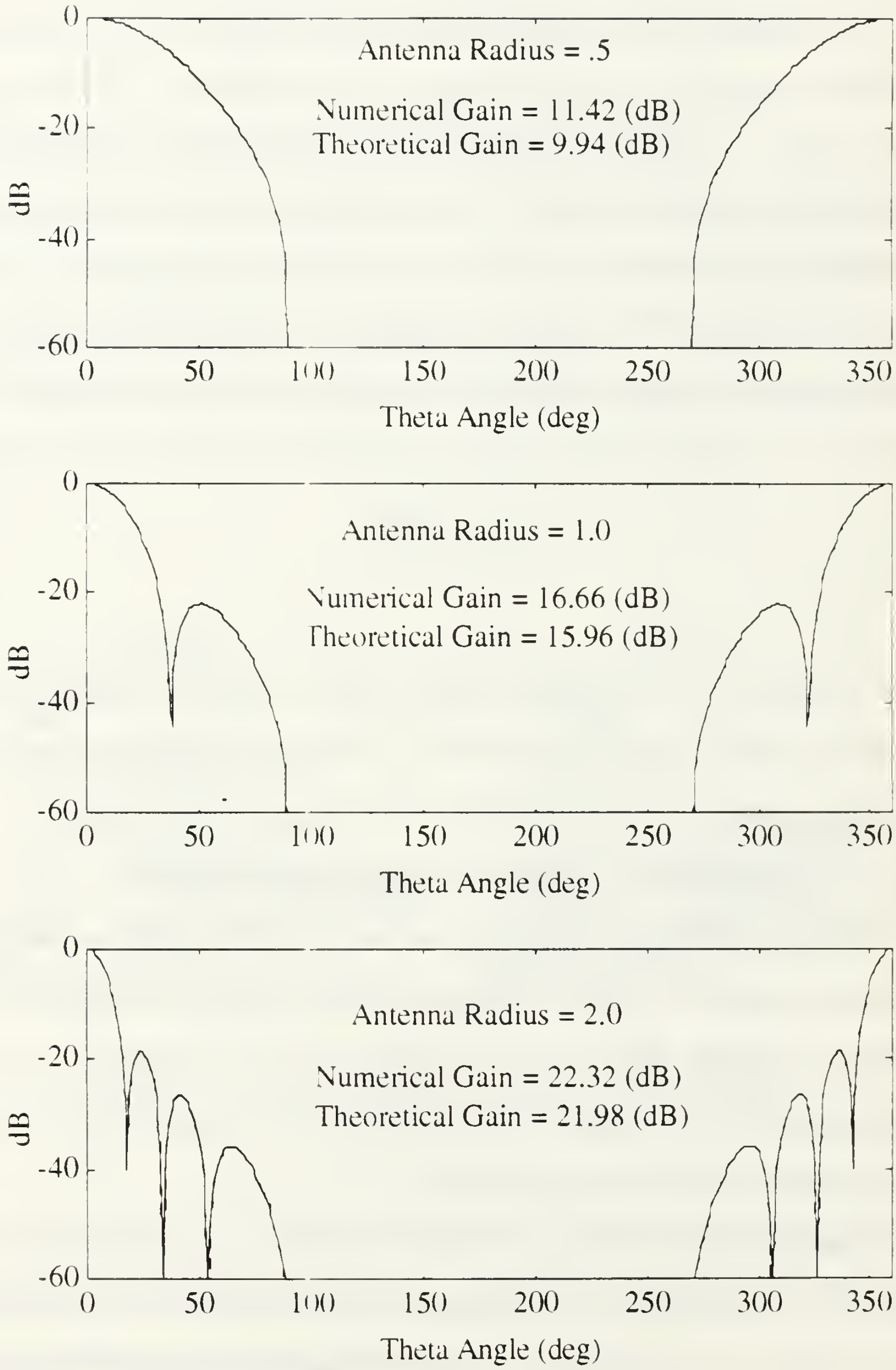

Figure 4.2 Scattering from a Negligibly Small Disk

computed by *RADOME.F* and *GAIN.F* are identical to that computed for the reflector.

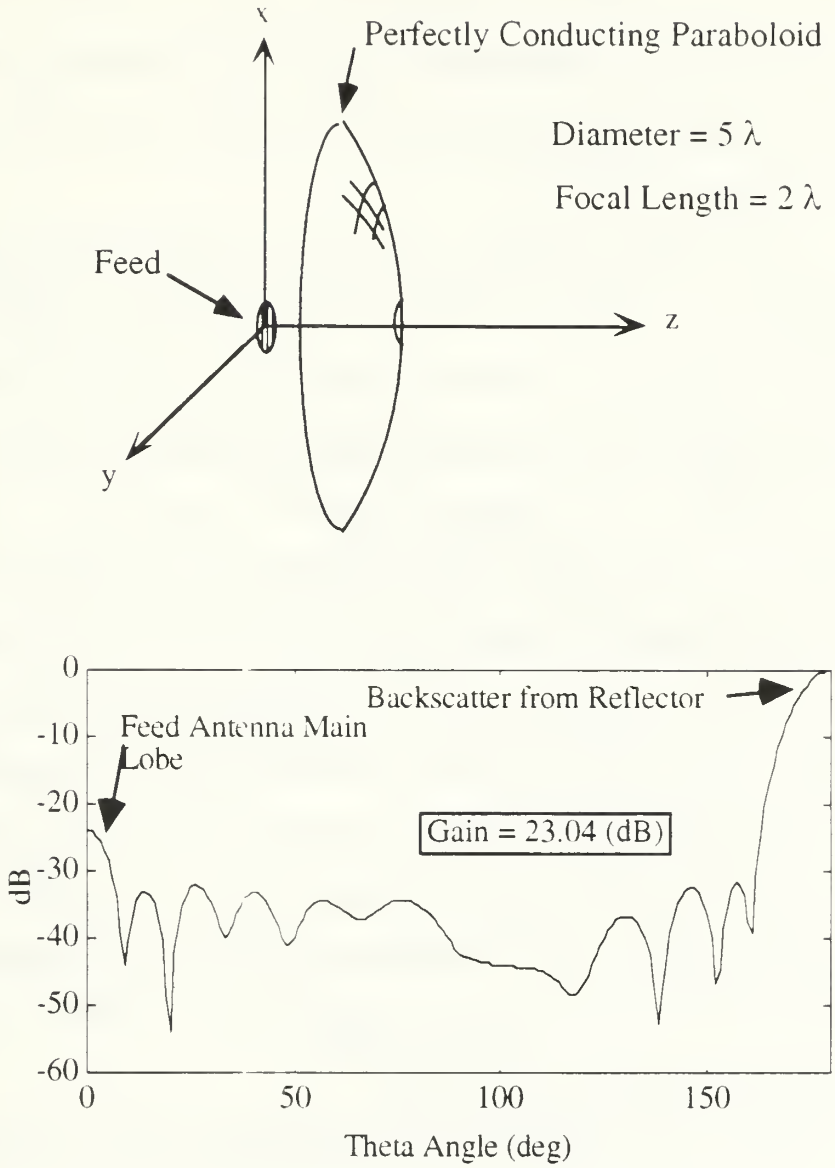

Figure 4.3 Perfectly Conducting Paraboloid

3. Dielectric Radomes

To determine the effect of various radome materials on antenna performance, radomes with several different surface impedances were analyzed.

Surface impedances were calculated using (2-32) for a range of ε_r and $\tan\delta$ typical of radome materials. The real and imaginary components of Z_l as a function of ε_r and $\tan\delta$ are plotted in Figure 4.4. As shown in the figure, the reactance is essentially independent of $\tan\delta$ over the range $0 \le \tan\delta \le 0.1$, and the real component of Z_l is strongly dependent on $\tan\delta$, particularly in the region of low ε_r.

A test radome consisting of an ogive with dimensions given in Table I and parameters $n_\lambda = 1/28$ and $\tan\delta = 0.01$ was analyzed.* The patterns were calculated for dielectric constants ranging $2 \le \varepsilon_r \le 10$, and the results are summarized in Table III. The E and H-plane patterns of an ogive radome with $Z_l = 34 - j1700$ Ω are shown in Figures 4.5 and 4.6. The impedance corresponds to a radome made out of a material of $\varepsilon_r = 2.0$ and $\tan\delta = 0.01$. The scattering is symmetric with respect to θ for the unscanned antenna, and the backlobe level is -45 dB below the peak. The scanned case has higher sidelobes, as expected, with a lobe of -30 dB at an angle $270°$. The gain loss due to the radome for the unscanned case is approximately 0.1 dB, and for the scanned case 0.2 dB. The discontinuity in the H-plane patterns at $\theta = 90°$ is due to the antenna model. When $\phi = 90°$ the field is ϕ polarized, and according to (2-71), E_ϕ^a has no $\cos\theta$ factor to force the pattern to 0 at $\theta = 90°$. Note that the radome ohmic loss has not been included.

Increasing the dielectric constant results in a constant-shape scattering pattern with increasing magnitude. Figure 4.7 is the E-field pattern for $\varepsilon_r = 10$, which corresponds to some ceramics. It is apparent that the scattering by the

---

* These dimensions are not intended to match any particular existing radome.

radome in the rear hemisphere is significant, and the gain loss is nearly 4 dB. Table III shows the gains with and without the radome for $2 \le \varepsilon_r \le 10$. Figure

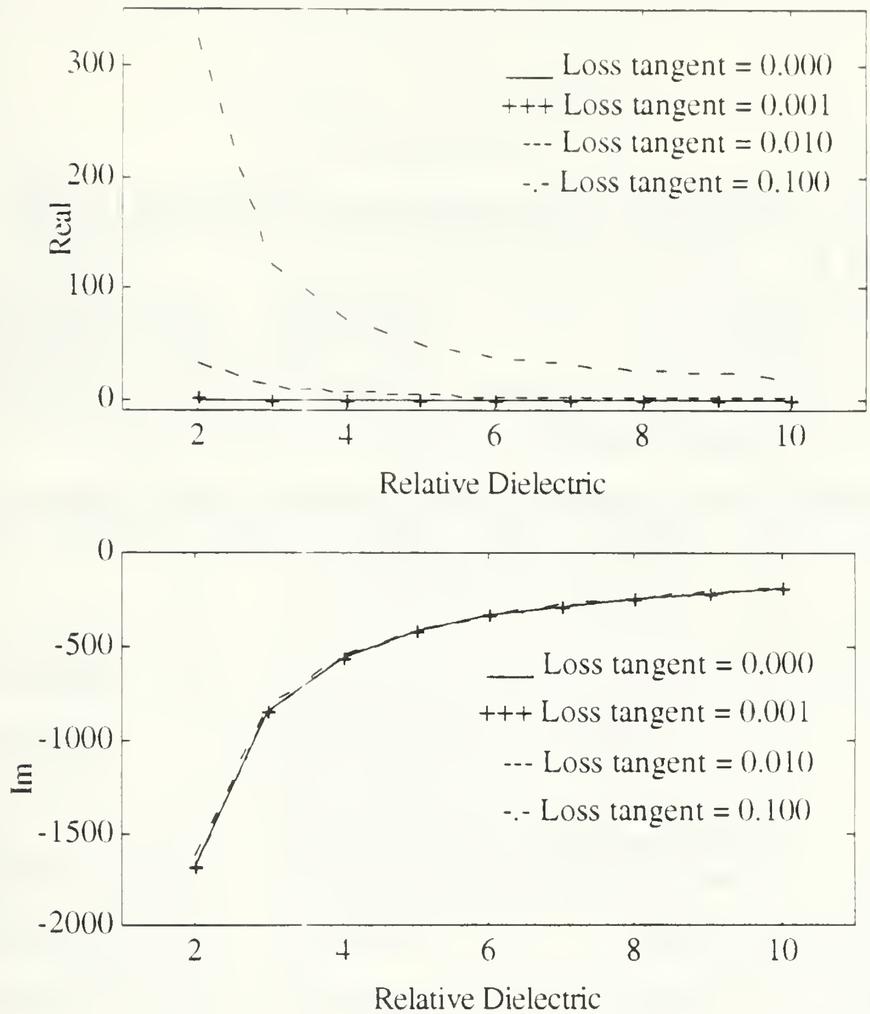

Figure 4.4 Surface Impedance Z_L as a Function of ε_r and $\tan\delta$

4.8 is a plot of the gain loss relative to no radome as a function of ε_r. For reference the Fresnel reflection coefficient for a planar interface is also shown.

TABLE III. GAIN LOSS FOR AN OGIVE RADOME

ε_r	Z_L (Ω) $\left(n_\lambda = \frac{1}{28},\ \tan\delta = 0.01\right)$	Gain (dB) With/Without Radome $\theta_s = 0^\circ$ $(\phi_s = 0^\circ)$	Gain (dB) With/Without Radome $\theta_s = 30^\circ$ $(\phi_s = 0^\circ)$
2.0	0-j1700	22.21/22.32	21.61/21.78
2.0	34-j1700	22.21/22.32	21.61/21.78
4.0	7.5-j560	21.42/22.32	20.90/21.78
5.0	5.3-j420	20.91/22.32	20.46/21.78
7.0	3.3-j280	19.95/22.32	19.47/21.78
8.0	2.7-j240	19.51/22.32	18.96/21.78
9.0	2.4-j210	19.04/22.32	18.47/21.78
10.0	2.1-j190	18.59/22.32	18.00/21.78

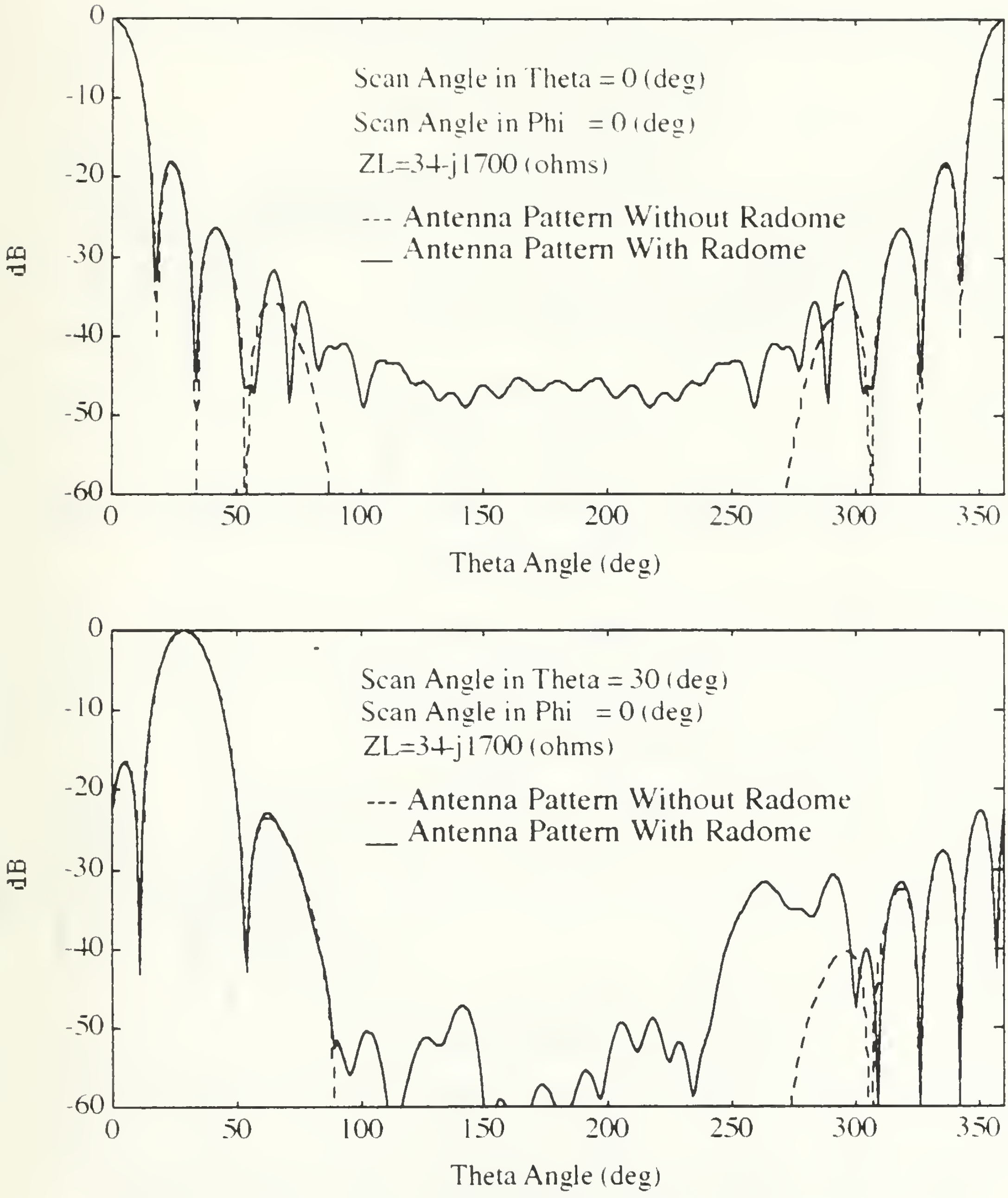

Figure 4.5 E-Plane Pattern of Antenna with an Ogive Radome
$Z_L = 34 - j1700\ \Omega$

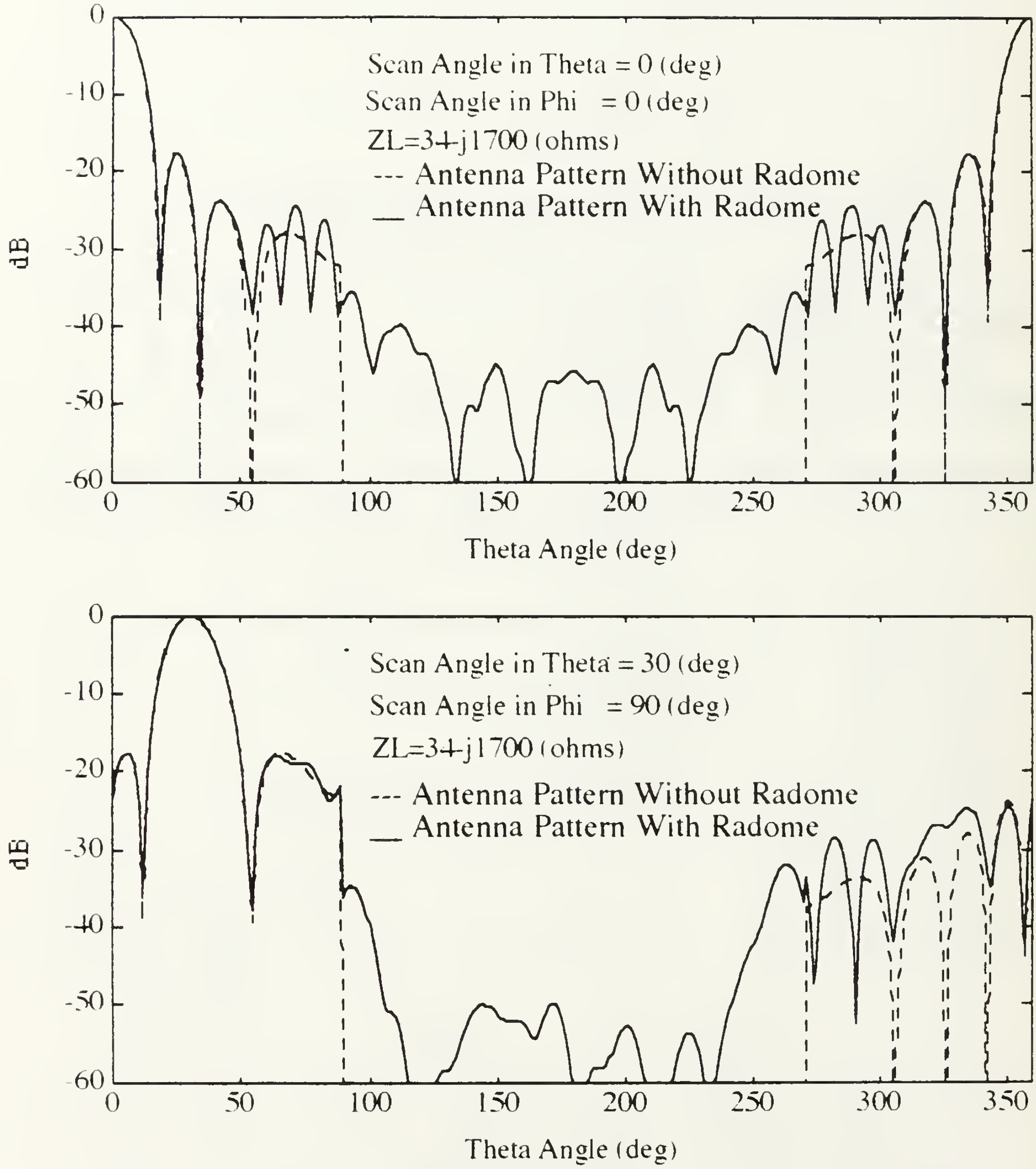

Figure 4.6 H-Plane Pattern of Antenna with an Ogive Radome
$Z_L = 34 - j1700\ \Omega$

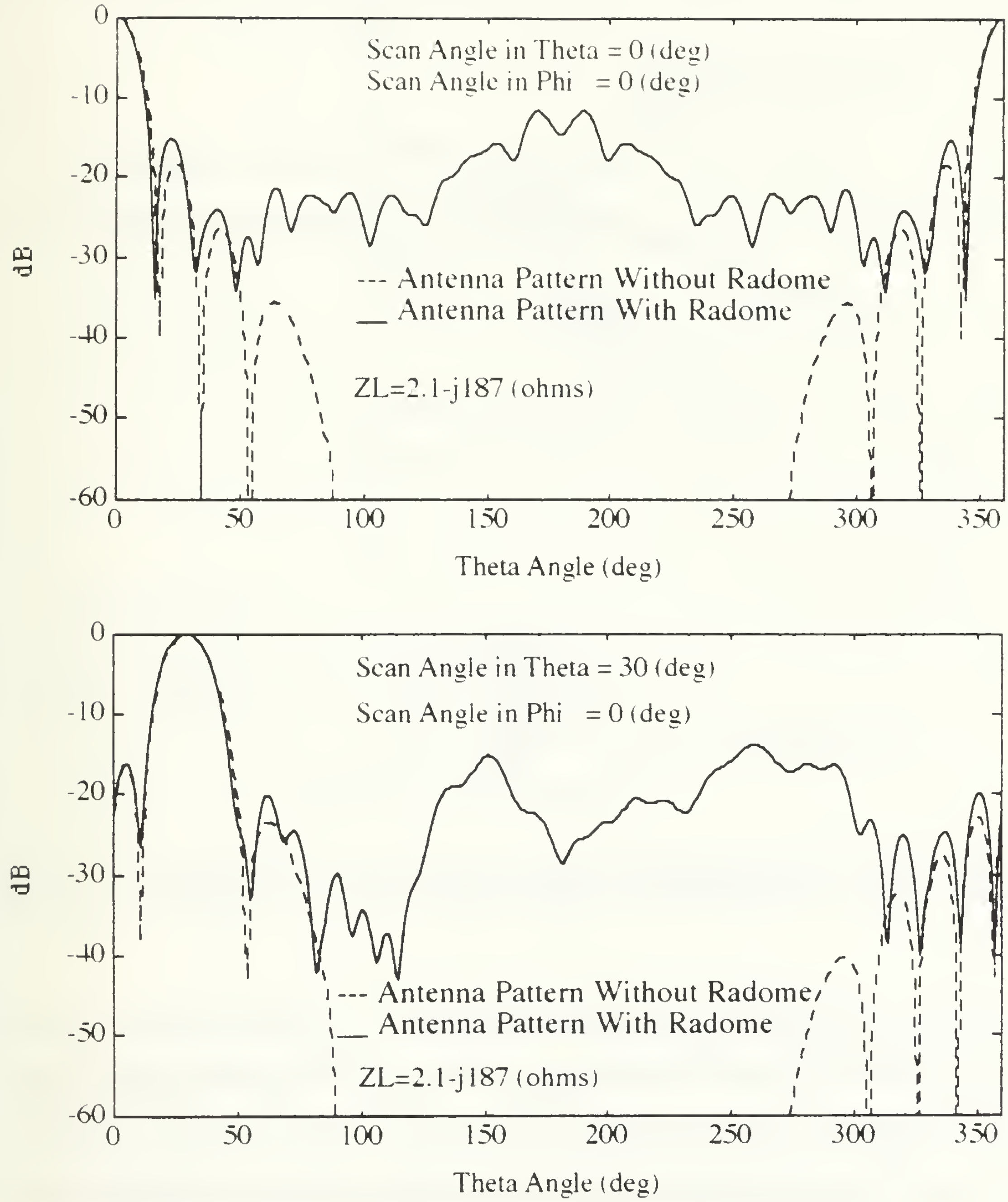

Figure 4.7 E-Plane Pattern of Antenna with an Ogive Radome
$Z_L = 2.1 - j187\ \Omega$

The gain loss appears to be linear with ε_r. This is probably because the base of the ogive is open, which allows reflected waves to exit the radome. If a back surface was present, these waves would be reflected again causing additional gain loss.

a. Conical Radome

To determine the effect that radome shape has on scattering, a conical radome was analyzed. The length, width, and impedance of the cone was

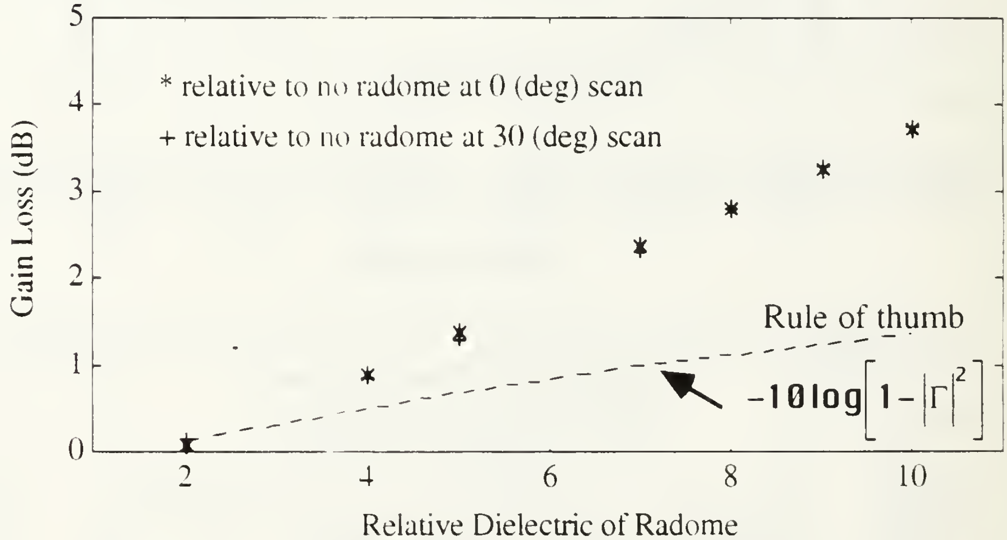

Figure 4.8 Gainloss Relative for an Ogive Radome as a Function of ε_r (E-Plane Scan)

the same as the ogive radome discussed previously. A conical radome presents a relatively flat surface that could possibly cause more focused scattering than a curved shape like an ogive. Ray tracing of Figure 4.9 shows that the reflections from the radome will add coherently for the cone, and non-coherently for the

ogive. However, this assumes that the radome is in the far field of the antenna. The E-plane pattern is shown in Figure 4.10.

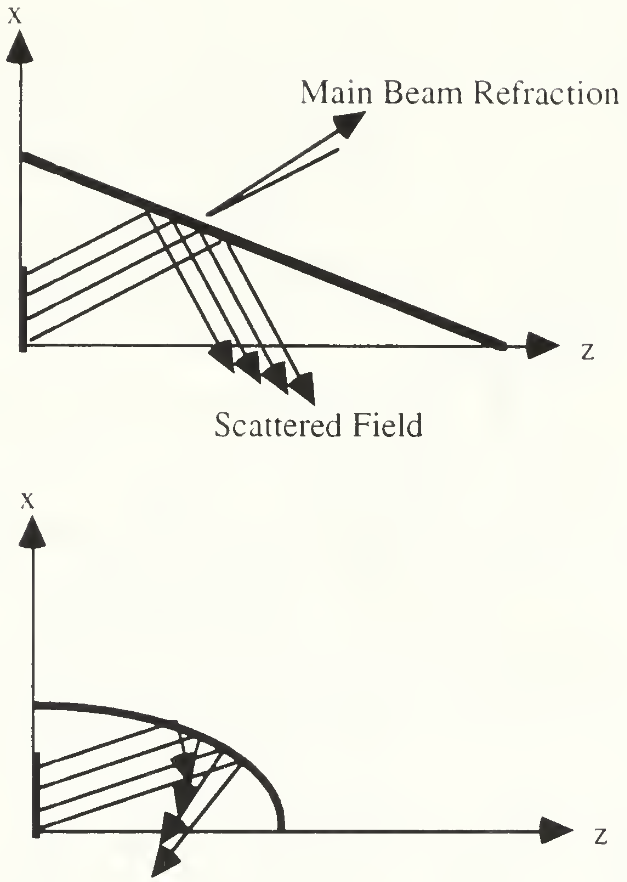

Figure 4.9 Scattering from Conical Radome vs. Ogive

A comparison of Figures 4.5 and 4.10 demonstrates that there is even less scattering from the conical radome in the rear hemisphere. For the scanned case, the conical radome has a distinct lobe at $\theta = 280°$ due to the specular reflection of the main lobe from one side of the radome. This is referred to as a reflection lobe.

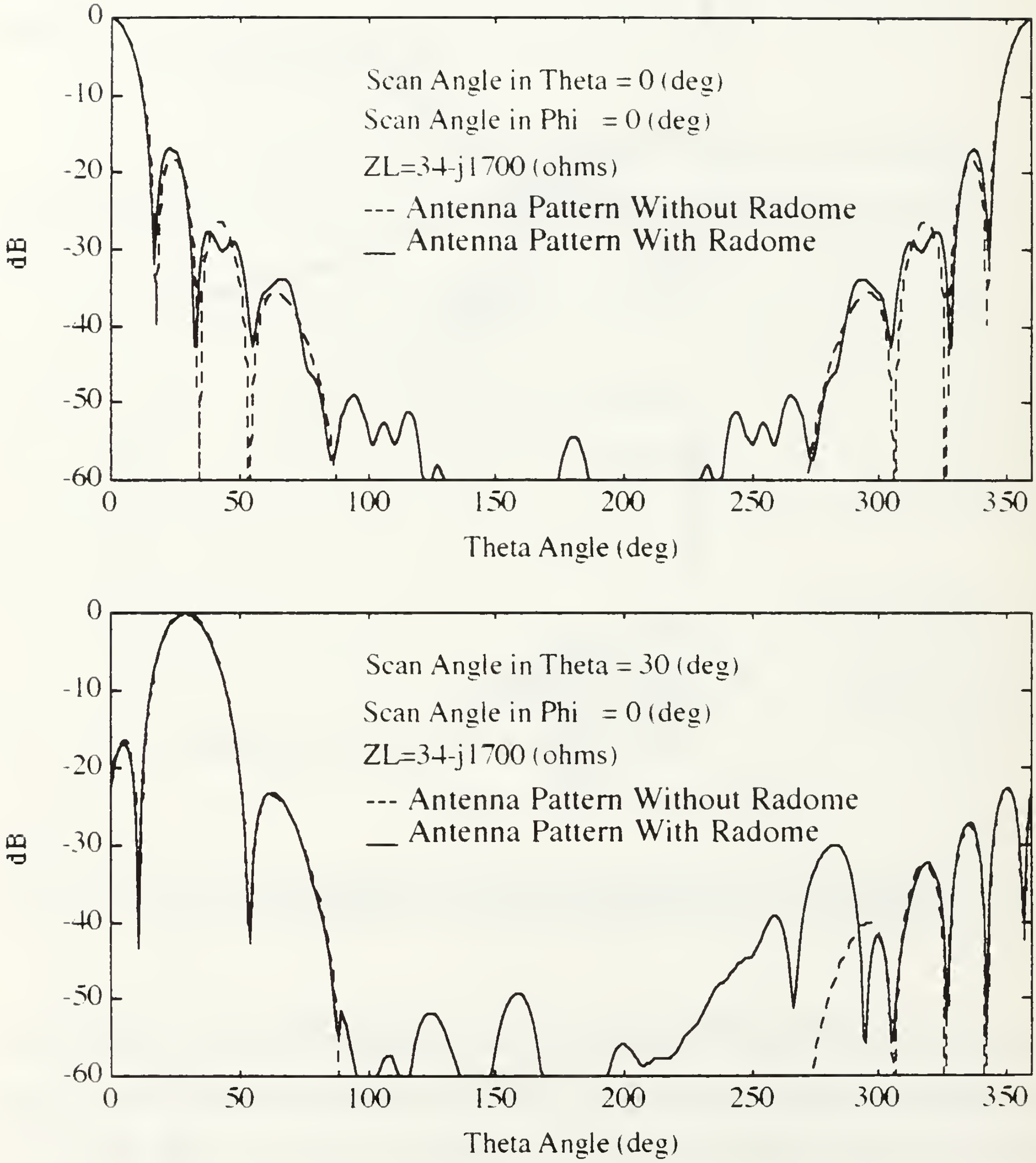

Figure 4.10 E-Pane Pattern of Antenna with a Conical Radome $Z_L = 34 - j1700\ \Omega$

B. CONCLUSIONS

This research has resulted in two computer codes that can be used to predict the pattern degradation and gain loss due to an axially symmetric radome in the near field of a linearly polarized circular aperture. The computed patterns show that the gain loss is slightly greater than the reflection loss for a planar interface with the same dielectric constant as the radome. This is approximately true for low-loss radomes. For lossy radomes the impedance has a very strong affect on scattering, while radome shape has less effect. Scanning the antenna results in an asymmetric scattering pattern and generally higher sidelobe levels.

The computed gain compares favorably with values computed by closed form solutions derived from theory, and the gain of the system has the expected decrease as the level of scattering increases. For the specific case of the ogive radome, there is a linear dependence of the gain loss to the relative dielectric of the radome. Note that the gain calculations presented here do not include ohmic loss inside of the radome.

Improvements in the original code include arbitrary aperture illumination and incorporating mode symmetry. Computational run time is reduced by nearly half when symmetry is used, and significantly increases the applicability of this method of analysis.

In order to apply this analysis to multilayer radomes, an equivalent surface impedance must be determined. The program can be modified to account for radomes constructed of nonuniform materials by assigning different impedances for each segment of the plane curve generating the BOR.

APPENDIX A. GAUSS QUADRATURE

Computing the MM elements of $\left[\mathbf{U}\right]$ and $\left[\mathbf{Z}\right]$ (2-19 and 2-28) as well as the gain requires the numerical evaluation of integrals. MM requires numerous integration in order to solve for the current coefficients I^{t}_{nj} and I^{o}_{nj} of (2-31). Not only is numerical analysis necessary to perform the integrals in the MM, but also the matrix analysis of (2-31). The large number of separate integrals necessitates an integration technique which provides accuracy in a few steps to avoid excessive run time. Gauss Quadrature is a method of numerical integration which employs unequally spaced intervals , and provides accuracy with a few points. [Ref. 4:pp. 154-157]

Gauss Quadrature approximation of the form

$$I = \int_{a}^{b} f(x)dx,, \tag{A-1}$$

by the sum of m terms

$$I = \frac{a-b}{2} \sum_{k=1}^{m} w_k f(x_k). \tag{A-2}$$

The coefficients w_k are weights, and the x_k's are abscissas determined from

$$x_k = \frac{b+a}{2} + \frac{b-a}{2} \xi_k, \tag{A-3}$$

where ξ_k's are the m number of zeros of the m'th degree Legendre Polynomials.

APPENDIX B. RADOME.F LISTING

The material on the following pages is a listing of the program *RADOME.F* described in chapter III.

```
C*************************************************************************
C  ROGRAM         :  radome.f   V.3
C  DATE           :  23 January 1992
C  LATEST REVISION:  31 December 1992
C  PROGRAMMERS    :  D. JENN, R. FRANCIS, K. KLOPP
C
C NOTES:
C   1. ARBITRARY CIRCULAR APERTURE ILLUMINATION SPECIFIED IN
C      FUNCTION TAPER.
C   2. NEAR FIELD COMPONENTS INCLUDED. (COMPUTED IN SUBROUTINE GENEX.)
C   3. ANTENNA IS AT THE BOR COORDINATE SYSTEM ORIGIN.
C   4. LINEARLY (X-POLARIZED) ANTENNA.
C   5. CLOSED FORM FAR FIELD ANTENNA OPTION ADDED.
C   6. FLAGS USED THROUGHOUT: (IMP IS SET AUTOMATICALLY)
C          IMP=0 perfect conductor radome (for test purposes)
C          IPRINT=0 print pattern points to unit 8
C          ISEG=0 print the generating curve points to unit 8
C          ICALC=0 compute current coefficients & pattern
C               #0 read current coefficients from disc file given
C                  by 'CURCO and compute the pattern.
C          IFF=0 closed form far field antenna pattern is used to
C                compute ETF and EPF (SUBROUTINE ANTFF)
C             #0 far field computed using CIRCRTP at distance ROBS
C          ISYM=0 use mode symmetry
C              #0 turn off mode symmetry (for testing)
C   7. THE FACTOR ETA (=377 OHMS) IS OMITTED THROUGHOUT.
C
C BEGIN MAIN PROGRAM*********************************************
      CHARACTER ITH
      CHARACTER*9 OUT,ETSCATTER,EPSCATTER,CURCO
      CHARACTER*12 CC
      CHARACTER*6 AN,ETFEED,EPFEED
      CHARACTER*8 GNT,GNPHI,CGP,CPOL,XPOL
      CHARACTER*14 TPTS,PPTS,PHIPTS
      COMPLEX EP,ET,Z(100000),R(1600),B(800),C(800),U
      COMPLEX UC,ET1,EP1,ET2,EP2,EC,EX,ZLO(400),ZL(2400)
      COMPLEX CIRCR,CIRCT,CIRCP,ETF,EPF
      COMPLEX EXP1,EXP2,CONJG,CEXP,CMPLX,CT(30000),IMP,JK
      DIMENSION RH(400),ZH(400),XT(4),AT(4),IPS(800),XP(100),AP(100)
      DIMENSION A(100),X(100),EXP(500),ANG(500),ECP(500)
      DIMENSION ECV(500),EXV(500),PHC(500),PHX(500)
      DIMENSION ETSCAT(500),EPSCAT(500),ETHF(500),EPHF(500)
      INTEGER CNPHI,SELECTION
      DATA IPRINT/0/,ICALC/1/,IFF/0/,ISEG/0/,ISYM/0/
      DATA START,STOP/0.,360./,PI/3.1415926/
      Rad=PI/180.
      ECX=0.
      BK=2.*PI
      U=(0.,1.)
      U0=(0.,0.)
      UC=-U/4./PI
```

```
      JK=(0.0,6.283185307)
C***** ENTER A LETTER TO INDICATE WHICH ITERATION THIS IS ***********
C*****   (ALL DATA FILES ARE APPENDED WITH THIS LETTER)   ***********
C*****   (.m FILES ARE FOR GENERATING PLOTS IN MATLAB)    ***********
        WRITE(6,*)'ENTER A LETTER TO INDICATE WHICH ITERATION THIS IS'
        READ(5,*)ITH
        OUT='outldbor'//ITH
        AN ='ang'//ITH//'.m'
        ETFEED='etf'//ITH//'.m'
        EPFEED='epf'//ITH//'.m'
        ETSCATTER='etscat'//ITH//'.m'
        EPSCATTER='epscat'//ITH//'.m'
        CURCO='asciidat'//ITH
        CC='curcoefsdat'//ITH
        CPOL='etpol'//ITH//'.m'
        XPOL='eppol'//ITH//'.m'
        IF(ICALC.EQ.0) THEN
      WRITE(6,*)'ENTER THE ANTENNA RADIUS (wavelengths)'
      READ(5,*) ARAD
C*****SELECT THE GEOMETRY OF THE BOR**************************************
           WRITE(6,*)'SELECT BOR GEOMETRY BY NUMBER.'
           WRITE(6,*)'   NUMBER          BOR'
           WRITE(6,*)'     1             OGIVE'
           WRITE(6,*)'     2             SPHERE'
           WRITE(6,*)'     3             CONE'
           WRITE(6,*)'     4             DISK'
           WRITE(6,*)'     5             PARABOLA'
           READ(5,*)SELECTION
           IF (SELECTION.EQ.1)THEN
             WRITE(6,*)'CALLING SUBROUTINE FOR OGIVE'
             CALL OGIVE(NP,ZH,RH,BASE,RS,ZP)
           ELSEIF(SELECTION.EQ.2)THEN
             WRITE(6,*)'CALLING SUBROUTINE FOR SPHERE'
             CALL TESTSPHERE(NP,ZH,RH,BASE,RS,ZP)
           ELSEIF(SELECTION.EQ.3)THEN
             WRITE(6,*)'CALLING SUBROUTINE FOR CONE'
             CALL CONE(NP,ZH,RH,BASE,RS,ZP)
           ELSEIF(SELECTION.EQ.4)THEN
             WRITE(6,*)'CALLING SUBROUTINE FOR DISK'
             CALL DISK(NP,ZH,RH,BASE,RS,ZP)
           ELSEIF(SELECTION.EQ.5)THEN
             WRITE(6,*)'CALLING SUBROUTINE FOR PARABOLA'
             CALL PARAB(NP,ZH,RH,DM,FOD,ZP)
             BASE=DM/2.
           ENDIF
          IF (NP.GT.400)THEN
            WRITE(6,*)'MAXIMUM NUMBER OF POINTS(NP) IS 400'
            GOTO 999
          ENDIF
      ENDIF
C*************************************************************
```

```
      WRITE(6,*)'ENTER THE FILENAMES gaus###'
      WRITE(6,*)'..........................'
      WRITE(6,*)'ENTER THE FILENAME IN T (NT)'
      READ(5,*)GNT
      WRITE(6,*)'ENTER THE FILENAME IN PHI (NPHI)'
      READ(5,*)GNPHI
      WRITE(6,*)'ENTER THE FILENAME IN X AND Y'
      READ(5,*)CGP
C OPEN THE FILES FOR THE gaus/gaus###
      TPTS='gaus/'//GNT
      PPTS='gaus/'//GNPHI
      PHIPTS='gaus/'//CGP
      OPEN(1,FILE=TPTS,STATUS='OLD')
      OPEN(2,FILE=PPTS,STATUS='OLD')
      OPEN(4,FILE=PHIPTS,STATUS='OLD')
      READ(1,*)NT
         IF(NT.GT.4)THEN
           WRITE(6,*)'MAXIMUM NUMBER OF POINTS(NT) IS 4'
           GOTO 999
         ENDIF
      READ(2,*)NPHI
         IF(NPHI.GT.200)THEN
           WRITE(6,*)'MAXIMUM NUMBER OF POINTS(NPHI) IS 200'
           GOTO 999
         ENDIF
      READ(4,*)CNPHI
         IF(CNPHI.GT.100)THEN
           WRITE(6,*)'MAXIMUM NUMBER OF POINTS(CNPHI) IS 100'
           GOTO 999
         ENDIF
C LOAD THE WEIGHTS AND ABSCISSAS IN THE VECTORS.
      DO 1 M=1,NT
         READ(1,*,END=1)XT(M),AT(M)
  1   CONTINUE
      DO 2 M=1,NPHI
          READ(2,*,END=2)X(M),A(M)
  2   CONTINUE
      DO 4 M=1,CNPHI
          READ(4,*,END=4)XP(M),AP(M)
  4   CONTINUE
      CLOSE(1)
      CLOSE(2)
      CLOSE(4)
      WRITE(6,*)'ENTER THE PLOTTING INCREMENT IN DEGREES'
      READ(5,*)DT
      IF(ICALC.EQ.0) THEN
       WRITE(6,*)'ENTER HIGHEST MODE'
       READ(5,*) MODES
       NBLOCK=2*MODES+1
       NROW=NBLOCK*(2*NP-3)
C CHECK FOR BOUNDS VIOLATION
```

```
        IF(NROW.GT.30000) THEN
          WRITE(6,*) 'BOUNDS VIOLATION FOR ARRAY CT(.)'
          GOTO 999
        ENDIF
      ENDIF
      WRITE(6,*)'ENTER PHI (observation) IN DEGREES'
      READ(5,*)P
      PHI=P*RAD
      IF(ICALC.EQ.0) THEN
      WRITE(6,*)'ENTER THE SCAN ANGLES IN DEGREES: THETA,PHI'
      READ(5,*) THD,PHD
      THS=THD*RAD
      PHS=PHD*RAD
      WRITE(6,*)'ENTER COMPLEX IMPEDANCE, OHMS: (Real,Imag)'
      READ(5,*) IMP
      ELSE
C**********READ CURRENT COEFFICIENTS IF ICALC#0************************
       WRITE(6,*) 'ENTER LETTER CODE FOR FILE curcoefsdat'
        READ(5,*) ITH
        CC='curcoefsdat'//ITH
       OPEN(31,FILE=CC)
       WRITE(6,*) 'FILE OPENED IS ',CC
       READ(31,*) ITH,SELECTION,BASE,NP,MT,MP,N
       READ(31,*) NROW,MODES,NBLOCK,THS,PHS,ARAD,PHI0
       READ(31,*) IMP,ZP,RS
       READ(31,*) (RH(L),L=1,NP)
       READ(31,*) (ZH(L),L=1,NP)
       READ(31,*) (ZLO(L),L=1,NP)
       READ(31,*) (CT(L),L=1,NROW)
       CLOSE(31)
       WRITE(6,*) 'RUN CODE READ FROM DISC : ',ITH
      ENDIF
C**********READY TO CALCULATE THE PATTERN*****************************
      IT=INT((STOP-START)/DT)+1
      MHI=MODES+1
      OPEN(8,FILE=OUT)
      WRITE(8,8000) 2.*BASE,NP,MODES,RS,ZP
 8000 FORMAT(//,'*** BOR RADIATION PATTERN FOR A CIRCULAR'
     *' DISC RADIATOR USING GENEX ***',
     *//,2X,'BOR DIAMETER (WAVELENGTHS)=',F5.2,/,2X,
     * 'NUMBER OF GENERATING POINTS (NP)=',I4,/,2X,
     * 'HIGHEST MODE NUMBER USED (MODES)=',I3,
     *//,2X,'SURFACE RADIUS',F5.2,/,2X,'ZPRIME',F5.2)
      WRITE(8,30) NT,NPHI,CNPHI
   30 FORMAT(/,12X,' NT   NPHI   CNPHI',
     *       /,10X,I5,2X,I5,2X,I5)
      IF(ISEG.EQ.0) WRITE(8,1300)
 1300 FORMAT(/,10X,'INDEX',8X,'Z(I)',10X,'RHO(I)',12X,'SURF IMPED')
      MP=NP-1
      MT=MP-1
      N=MT+MP
```

```
      DO 52 I=1,NP
      IF(ABS(ZH(I)).LT..001) ZH(I)=0.
      IF(ABS(RH(I)).LT..001) RH(I)=0.
      ZHB=ZH(I)/BK
      RHB=RH(I)/BK
C ASSIGN SURFACE IMPEDANCE AT THIS POINT.  THE SURFACE IMPEDANCE OF SEGMENT
C I IS ZLO(I)
      IF(ICALC.EQ.0) Z1O(I)=IMP/(120.*PI)
      IF(ISEG.EQ.0) WRITE(8,8004) I,ZHB,RHB,IMP
52    CONTINUE
8004  FORMAT(11X,I4,4X,F8.3,8X,F8.3,6X,2F8.2)
C***********************************************************************
C BIG LOOP  * > * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C MODE LOOP TO CALCULATE THE CURRENT COEFFICIENTS. POS AND NEG MODES    *
C DONE IN THE SAME ITERATION OF THE LOOP
C ^                                                                     v
C 400 CONTINUE <  * * * * * * * * * * * * * * * * * * * * * * * * * * *
C**********************************************ZLOAD,ZMAT,GENEX,DECOMP,SOLVE
      IF(ICALC.EQ.0) THEN
      IF(CABS(IMP).NE.0) CALL ZLOAD(NP,RH,ZH,ZLO,ZL)
      DO 400 M=1,MHI
      NM=M-1
      CALL ZMAT(NM,NM,NP,NPHI,NT,RH,ZH,X,A,XT,AT,Z)
C*******************************************************POSITIVE MODE +n
C          |  t,t      t,phi   |
C          | Z         Z       |
C          | +n        +n      |
C  ZMAT =  |                   |
C          |  phi,t    phi,phi |
C          | Z         Z       |
C          | +n        +n      |
C**********************************************************************
C  MODE SYMMETRY IN n FOR ZMAT EXISTS BUT IS NOT USED IN THIS VERSION.
C  MODE SYMMETRY IN I USED DIRECTLY FOR PHIS OF 0,90,180, AND 270.
C**********************************************************************
C   |  t,t      t,phi   |   |  t,t      t,phi   |
C   | Z         Z       |   | Z        -Z       |
C   | -n        -n      |   | +n        +n      |
C   |                   | = |                   |
C   |  phi,t    phi,phi |   |  phi,t    phi,phi |
C   | Z         Z       |   | -Z        Z       |
C   | -n        -n      |   | +n        +n      |
C****************************************************CURRENT COEFFICIENTS
C                        ------ ------ ------ -1
C         | _  |   |      |      |      |    | _   |
C         | I  |   | Z    |  0   |  0   |    | V   |
C         |  +n|   |  +n  |      |      |    |  +n |
C         |    |   |______|______|______|    |     |
C         | _  |   |      |      |      |    | _   |
C   CT =  | I  | = |   0  | Z    |  0   |  * | V   |
```

```
C          |   0|   |      |  0  |      |    |  0  |
C          |    |   |______|_____|______|    |     |
C          | _  |   |      |     |      |    | _   |
C          | I  |   |  0   |  0  |  Z   |    | V   |
C          |  -n|   |      |     |   -n |    |  -n |
C          |    |   |______|_____|______|    |     |
C****************************************************POSITIVE MODE +n
      IF(CABS(IMP).NE.0) CALL ZTOT(MT,MP,ZL,Z)
      CALL GENEX(NM,NP,NT,NPHI,CNPHI,XT,AT,X,A,
     *           XP,AP,THS,PHS,ARAD,RH,ZH,B)
      CALL DECOMP(N,IPS,Z)
      CALL SOLVE(N,IPS,Z,B,C)
      NTOP1=MODES-NM
      NTOP2=NBLOCK-(NTOP1+1)
      NS2=NTOP1*N
      NS1=NTOP2*N
      NROW=NBLOCK*N
C*****************************************************************
C STORE CURRENT COEFFICIENTS IN ONE LONG VECTOR CT(.). MOST NEGATIVE
C MODE IS AT THE TOP; MOST POSITIVE AT THE BOTTOM.  UNIT 30 IS FORMATED
C OUTPUT; UNIT 31 IS FREE FORMATED (USED BY GAIN PROGRAM).
C*****************************************************************
         OPEN(30,FILE=CURCO)
         WRITE(30,5027)'Mode(',NM,') = ',NM,'NM = ',NM
      DO 401 L=1,N
         WRITE(30,5026) 'CT(',L+ NS1,') = ',C(L)
401   CT(L+NS1)=C(L)
5027  FORMAT(/A,I,A,I/A,I/)
5026  FORMAT(5X,A,I4,A,F10.4,'+ (',F10.4,')*i')
C***************END OF POSITIVE MODE*******BEGIN NEGATIVE MODE -n
C MODE SYMMETRY IS USED IF ISYM=0
      IF(NM.NE.0) THEN
      NMN=-NM
C USE BRUTE FORCE IF ISYM # 0
      IF(ISYM.NE.0) THEN
      CALL GENEX(NMN,NP,NT,NPHI,CNPHI,XT,AT,X,A,
     *           XP,AP,THS,PHS,ARAD,RH,ZH,B)
      CALL ZMAT(NMN,NMN,NP,NPHI,NT,RH,ZH,X,A,XT,AT,Z)
      IF(CABS(IMP).NE.0) CALL ZTOT(MT,MP,ZL,Z)
      CALL DECOMP(N,IPS,Z)
      CALL SOLVE(N,IPS,Z,B,C)
      ENDIF
        WRITE(30,5027)'Mode(',NMN,') = ',NMN,'NM = ',NM
        DO 402 L=1,MT
        CT(L+NS2)=+C(L)
        WRITE(30,5026) 'CT(',L+NS2,') = ',CT(L+NS2)
402     CONTINUE
        DO 403 L=MT+1,N
        IF(ISYM.NE.0) CT(L+NS2)=+C(L)
        IF(ISYM.EQ.0) CT(L+NS2)=-C(L)
        WRITE(30,5026) 'CT(',L+NS2,') = ',CT(L+NS2)
```

```
403     CONTINUE
C*******************************************END OF NEGATIVE MODE -n
      ENDIF
400   CONTINUE
      CLOSE(30)
C***********************************************************************
C WRITE THE VECTOR OF CURRENT COEFFICIENTS ON DISC FOR GAIN CALC.
C (FREE FORMAT TO UNIT 31.)
C***********************************************************************
       OPEN(31,FILE=CC)
       WRITE(31,*) ITH,SELECTION,BASE,NP,MT,MP,N
       WRITE(31,*) NROW,MODES,NBLOCK,THS,PHS,ARAD,PHI
       WRITE(31,*) IMP,ZP,RS,IFF
       WRITE(31,*) (RH(L),L=1,NP)
       WRITE(31,*) (ZH(L),L=1,NP)
       WRITE(31,*) (ZLO(L),L=1,NP)
       WRITE(31,*) (CT(L),L=1,NROW)
       WRITE(31,*) GNT,GNPHI,CGP
       WRITE(6,*) 'CURRENT COEFFICIENTS WRITTEN ON DISC'
       CLOSE(31)
       ENDIF
C************************************************************************
C END OF BIG LOOP   * > * * * * * * * * * * * * * * * * * * * * * * * * *
C ^                                                                     v
C * * * * * * * * <  * * * * * * * * * * * *  * * * * * * * * * * * * * *
C************************************************************************
C
C BEGIN PATTERN LOOP FROM START TO STOP IN INCREMENTS OF DT (ALL IN DEG)
C
      DO 500 I=1,IT
       THETA=START + DT*FLOAT(I-1)
       THX=THETA*RAD
       PHR=PHI
      IF(THETA.GT.180.) THEN
      PHR=PHI+PI
      THX=(360.-THETA)*RAD
      ENDIF
      ET1=(0.,0.)
      EP1=(0.,0.)
      ET2=(0.,0.)
      EP2=(0.,0.)
      DO 300 M=1,MHI
      NM=M-1
      EXP1=CEXP(CMPLX(0.,FLOAT(NM)*PHR))
      EXP2=CONJG(EXP1)
C*************************************************************PLANE
      CALL PLANE(NM,NM,NP,NT,RH,ZH,XT,AT,THX,R)
      NTOP1=MODES-NM
      NTOP2=NBLOCK-(NTOP1+1)
      NS2=NTOP1*N
      NS1=NTOP2*N
```

```
      DO 250 L=1,MT
      ET1=ET1+R(L)*CT(L+NS1)*EXP1
      EP1=EP1+R(L+N)*CT(L+NS1)*EXP1
      IF(NM.EQ.0) GOTO 250
      ET1=ET1+R(L)*CT(L+NS2)*EXP2
      EP1=EP1-R(L+N)*CT(L+NS2)*EXP2
250   CONTINUE
      DO 260 L=1,MP
      ET2=ET2+R(L+MT)*CT(L+NS1+MT)*EXP1
      EP2=EP2+R(L+MT+N)*CT(L+NS1+MT)*EXP1
      IF(NM.EQ.0) GO TO 260
      ET2=ET2-R(L+MT)*CT(L+NS2+MT)*EXP2
      EP2=EP2+R(L+MT+N)*CT(L+NS2+MT)*EXP2
260   CONTINUE
300   CONTINUE
C FEED CONTRIBUTION IN THE FAR FIELD IS ETF,EPF
C*****************************************************BESSJ1.
C USE CLOSED FORM FEED EXPRESSION FROM SUBROUTINE ANTFF IF
C IFF=0; OTHERWISE USE BRUTE FORCE EVALUATION FROM CIRCRTP
C
      ROBS=1000.
      IF(IFF.EQ.0) THEN
       CALL ANTFF(THX,PHR,THS,PHS,ARAD,ETF,EPF)
      ELSE
       RHB=ROBS*SIN(THX)
       ZHB=ROBS*COS(THX)
       CALL CIRCRTP(CNPHI,XP,AP,ARAD,THS,PHS,
     *                     PHR,RHB,ZHB,CIRCR,CIRCT,CIRCP)
C REMOVE THE 1/R DEPENDENCE BECAUSE EXP(-jkR)/R IS OMITTED IN
C THE SCATTERED FIELDS ET AND EP
       ETF=CIRCT*ROBS
       EPF=CIRCP*ROBS
      ENDIF
C*****************************************************
      ETHF(I)=CABS(ETF)
      EPHF(I)=CABS(EPF)
      ET=(ET1+ET2)*UC
      EP=(EP1+EP2)*UC
      ETSCAT(I)=CABS(ET)
      EPSCAT(I)=CABS(EP)
C TOTAL E-THETA AND E-PHI COMPONENTS
      EC=ET+ETF
      EX=EP+EPF
      ECV(I)=CABS(EC)
      EXV(I)=CABS(EX)
      ECR=REAL(EC)
      ECI=AIMAG(EC)
      EXR=REAL(EX)
      EXI=AIMAG(EX)
      PHC(I)=ATAN2(ECI,ECR+1.e-10)/RAD
      PHX(I)=ATAN2(EXI,EXR+1.e-10)/RAD
```

```
      ANG(I)=THETA
      ECX=AMAX1(ECX,ECV(I),EXV(I))
  500 CONTINUE
      WRITE(6,*) 'MAX E VALUE=',ECX
      WRITE(8,103) P,THS/RAD,PHS/RAD,ARAD,ECX
  103 FORMAT(/,10X,'PHI OF RECEIVER (DEG)=',F8.2,/,10X,
     *             'ANTENNA SCAN: THETA (DEG)=',F8.2,/,10X,
     *             '               PHI   (DEG)=',F8.2,/,10X,
     *             'ANTENNA RADIUS:      ARAD=',F8.3,/,10X,
     * 'MAXIMUM FIELD VALUE (V/M)=',E15.5)
      IF(IFF.EQ.0) THEN
       WRITE(8,113)
      ELSE
       WRITE(8,213) ROBS
      ENDIF
  113 FORMAT(/,10X,'CLOSED FORM FAR FIELD PATTERN FROM ANTFF IS USED')
  213 FORMAT(/,10X,'FAR FIELD COMPUTED USING CIRCRTP, ROBS=',F10.2)
C STORE DATA FOR NORMALIZED CO- AND CROS-POLARIZED PATTERNS
      DO 600 I=1,IT
      ECP(I)=(ECV(I)/ECX)**2
      EXP(I)=(EXV(I)/ECX)**2
      ECP(I)=AMAX1(ECP(I),1.E-6)
      EXP(I)=AMAX1(EXP(I),1.E-6)
      ECP(I)=10.*ALOG10(ECP(I))
      EXP(I)=10.*ALOG10(EXP(I))
  600 CONTINUE
      IF(IPRINT.EQ.0) THEN
      WRITE(8,5015)
 5015 FORMAT(//,7X,'ANGLE',15X,'CO-POL',25X,'X-POL',/,7X,
     1'(DEG)',4X,2('(VOLTS)',4X,'(DEG)',3X,'(DB-REL)',4X))
      DO 9000 L=1,IT
      WRITE(8,5016) ANG(L),ECV(L),PHC(L),ECP(L),EXV(L),PHX(L)
     1,EXP(L)
 5016 FORMAT(5X,F6.2,3X,2(F8.4,3X,F7.2,3X,F7.2,3X))
 9000 CONTINUE
      ENDIF
      OPEN(2,file=AN)
      OPEN(3,file=CPOL)
      OPEN(4,file=XPOL)
      OPEN(7,file=ETSCATTER)
      OPEN(8,file=EPSCATTER)
      OPEN(9,file=ETFEED)
      OPEN(10,file=EPFEED)
      DO 9097 I=1,IT
      WRITE(2,5019)I, ANG(I)
      WRITE(3,5020)I, ECP(I)
      WRITE(4,5021)I, EXP(I)
      WRITE(7,5022)I, ETSCAT(I)
      WRITE(8,5023)I, EPSCAT(I)
      WRITE(9,5024)I, ETHF(I)
 9097 WRITE(10,5025)I, EPHF(I)
```

```
 5019 FORMAT('ANG(',I,')=',F8.3,';')
 5020 FORMAT('ECP(',I,')=',F8.3,';')
 5021 FORMAT('EXP(',I,')=',F8.3,';')
 5022 FORMAT('ETSCAT(',I,')=',F8.3,';')
 5023 FORMAT('EPSCAT(',I,')=',F8.3,';')
 5024 FORMAT('ETHF(',I,')=',F8.3,';')
 5025 FORMAT('EPHF(',I,')=',F8.3,';')
      CLOSE(2)
      CLOSE(3)
      CLOSE(4)
      CLOSE(7)
      CLOSE(8)
      CLOSE(9)
      CLOSE(10)
 999  CONTINUE
      STOP
      END
C**********************************************************END OF MAIN PROGRAM.
C
C**********************************************************SUBROUTINE ZMAT.
C REFERENCE: AN IMPROVED E-FIELD SOLUTION FOR CONDUCTING BOR
C            J.R.MAUTZ AND  R.F. HARRINGTON
C            TECHNICAL REPORT TR-80-1
C            ROME AIR DEVELOPMENT CENTER
C            CONTRACT NO. F-30602-79-C-0011
      SUBROUTINE ZMAT(M1,M2,NP,NPHI,NT,RH,ZH,X,A,XT,AT,Z)
C
C COMPUTE THE MM IMPEDANCE MATRIX ELEMENTS.  THIS IS FROM MAUTZ AND
C HARRINGTON (NO CHANGES).
C
      COMPLEX Z(100000),U1,U2,U3,U4,U5,U6,U7,U8,U9,UA,UB,G4A(4),G5A(4)
      COMPLEX G6A(4),G4B(4),G5B(4),G6B(4),H4A,H5A,H6A,H4B,H5B
      COMPLEX H6B,UC,UD,GA(100),GB(100)
      DIMENSION RH(400),ZH(400),X(100),A(100),XT(4),AT(4)
      DIMENSION RS(400),ZS(400),D(400),DR(400),DZ(400)
      DIMENSION DM(400),C2(100),C3(100),R2(4),Z2(4)
      DIMENSION C4(100),C5(100),C6(100),Z7(4),R7(4),Z8(4),R8(4)
      CT=2.
      CP=.1
      DO 10 I=2,NP
      I2=I-1
      RS(I2)=.5*(RH(I)+RH(I2))
      ZS(I2)=.5*(ZH(I)+ZH(I2))
      D1=.5*(RH(I)-RH(I2))
      D2=.5*(ZH(I)-ZH(I2))
      D(I2)=SQRT(D1*D1+D2*D2)
      DR(I2)=D1
      DZ(I2)=D2
      IF(RS(I2).EQ.0.) RS(I2)=1.
      DM(I2)=D(I2)/RS(I2)
   10 CONTINUE
```

```
      M3=M2-M1+1
      M4=M1-1
      PI2=1.570796
      DO 11 K=1,NPHI
      PH=PI2*(X(K)+1.)
      C2(K)=PH*PH
      SN=SIN(.5*PH)
      C3(K)=4.*SN*SN
      A1=PI2*A(K)
      D4=.5*A1*C3(K)
      D5=A1*COS(PH)
      D6=A1*SIN(PH)
      M5=K
      DO 29 M=1,M3
      PHM=(M4+M)*PH
      A2=COS(PHM)
      C4(M5)=D4*A2
      C5(M5)=D5*A2
      C6(M5)=D6*SIN(PHM)
      M5=M5+NPHI
   29 CONTINUE
   11 CONTINUE
      MP=NP-1
      MT=MP-1
      N=MT+MP
      N2N=MT*N
      N2=N*N
      U1=(0.,.5)
      U2=(0.,2.)
      JN=-1-N
      DO 15 JQ=1,MP
      KQ=2
      IF(JQ.EQ.1) KQ=1
      IF(JQ.EQ.MP) KQ=3
      R1=RS(JQ)
      Z1=ZS(JQ)
      D1=D(JQ)
      D2=DR(JQ)
      D3=DZ(JQ)
      D4=D2/R1
      D5=DM(JQ)
      SV=D2/D1
      CV=D3/D1
      T6=CT*D1
      T62=T6+D1
      T62=T62*T62
      R6=CP*R1
      R62=R6*R6
      DO 12 L=1,NT
      R2(L)=R1+D2*XT(L)
      Z2(L)=Z1+D3*XT(L)
```

```
12 CONTINUE
   U3=D2*U1
   U4=D3*U1
   DO 16 IP=1,MP
   R3=RS(IP)
   Z3=ZS(IP)
   R4=R1-R3
   Z4=Z1-Z3
   FM=R4*SV+Z4*CV
   PHM=ABS(FM)
   PH=ABS(R4*CV-Z4*SV)
   D6=PH
   IF(PHM.LE.D1) GO TO 26
   D6=PHM-D1
   D6=SQRT(D6*D6+PH*PH)
26 IF(IP.EQ.JQ) GO TO 27
   KP=1
   IF(T6.GT.D6) KP=2
   IF(R6.GT.D6) KP=3
   GO TO 28
27 KP=4
28 GO TO (41,42,41,42),KP
42 DO 40 L=1,NT
   D7=R2(L)-R3
   D8=Z2(L)-Z3
   Z7(L)=D7*D7+D8*D8
   R7(L)=R3*R2(L)
   Z8(L)=.25*Z7(L)
   R8(L)=.25*R7(L)
40 CONTINUE
   Z4=R4*R4+Z4*Z4
   R4=R3*R1
   R5=.5*R3*SV
   DO 33 K=1,NPHI
   A1=C3(K)
   RR=Z4+R4*A1
   UA=0.
   UB=0.
   IF(RR.LT.T62) GO TO 34
   DO 35 L=1,NT
   R=SQRT(Z7(L)+R7(L)*A1)
   SN=-SIN(R)
   CS=COS(R)
   UC=AT(L)/R*CMPLX(CS,SN)
   UA=UA+UC
   UB=XT(L)*UC+UB
35 CONTINUE
   GO TO 36
34 DO 37 L=1,NT
   R=SQRT(Z8(L)+R8(L)*A1)
   SN=-SIN(R)
```

```
      CS=COS(R)
      UC=AT(L)/R*SN*CMPLX(-SN,CS)
      UA=UA+UC
      UB=XT(L)*UC+UB
   37 CONTINUE
      A2=FM+R5*A1
      D9=RR-A2*A2
      R=ABS(A2)
      D7=R-D1
      D8=R+D1
      D6=SQRT(D8*D8+D9)
      R=SQRT(D7*D7+D9)
      IF(D7.GE.0.) GO TO 38
      A1=ALOG((D8+D6)*(-D7+R)/D9)/D1
      GO TO 39
   38 A1=ALOG((D8+D6)/(D7+R))/D1
   39 UA=A1+UA
      UB=A2*(4./(D6+R)-A1)/D1+UB
   36 GA(K)=UA
      GB(K)=UB
   33 CONTINUE
      K1=0
      DO 45 M=1,M3
      H4A=0.
      H5A=0.
      H6A=0.
      H4B=0.
      H5B=0.
      H6B=0.
      DO 46 K=1,NPHI
      K1=K1+1
      D6=C4(K1)
      D7=C5(K1)
      D8=C6(K1)
      UA=GA(K)
      UB=GB(K)
      H4A=D6*UA+H4A
      H5A=D7*UA+H5A
      H6A=D8*UA+H6A
      H4B=D6*UB+H4B
      H5B=D7*UB+H5B
      H6B=D8*UB+H6B
   46 CONTINUE
      G4A(M)=H4A
      G5A(M)=H5A
      G6A(M)=H6A
      G4B(M)=H4B
      G5B(M)=H5B
      G6B(M)=H6B
   45 CONTINUE
      IF(KP.NE.4) GO TO 47
```

```
      A2=D1/(PI2*R1)
      D6=2./D1
      D8=0.
      DO 63 K=1,NPHI
      A1=R4*C2(K)
      R=R4*C3(K)
      IF(R.LT.T62) GO TO 64
      D7=0.
      DO 65 L=1,NT
      D7=D7+AT(L)/SQRT(Z7(L)+A1)
   65 CONTINUE
      GO TO 66
   64 A1=A2/(X(K)+1.)
      D7=D6*ALOG(A1+SQRT(1.+A1*A1))
   66 D8=D8+A(K)*D7
   63 CONTINUE
      A1=.5*A2
      A2=1./A1
      D8=-PI2*D8+2./R1*(BLOG(A2)+A2*BLOG(A1))
      DO 67 M=1,M3
      G5A(M)=D8+G5A(M)
   67 CONTINUE
      GO TO 47
   41 DO 25 M=1,M3
      G4A(M)=0.
      G5A(M)=0.
      G6A(M)=0.
      G4B(M)=0.
      G5B(M)=0.
      G6B(M)=0.
   25 CONTINUE
      DO 13 L=1,NT
      A1=R2(L)
      R4=A1-R3
      Z4=Z2(L)-Z3
      Z4=R4*R4+Z4*Z4
      R4=R3*A1
      DO 17 K=1,NPHI
      R=SQRT(Z4+R4*C3(K))
      SN=-SIN(R)
      CS=COS(R)
      GA(K)=CMPLX(CS,SN)/R
   17 CONTINUE
      D6=0.
      IF(R62.LE.Z4) GO TO 51
      DO 62 K=1,NPHI
      D6=D6+A(K)/SQRT(Z4+R4*C2(K))
   62 CONTINUE
      Z4=3.141593/SQRT(Z4/R4)
      D6=-PI2*D6+ALOG(Z4+SQRT(1.+Z4*Z4))/SQRT(R4)
   51 A1=AT(L)
```

```
      A2=XT(L)*A1
      K1=0
      DO 30 M=1,M3
      U5=0.
      U6=0.
      U7=0.
      DO 32 K=1,NPHI
      UA=GA(K)
      K1=K1+1
      U5=C4(K1)*UA+U5
      U6=C5(K1)*UA+U6
      U7=C6(K1)*UA+U7
   32 CONTINUE
      U6=D6+U6
      G4A(M)=A1*U5+G4A(M)
      G5A(M)=A1*U6+G5A(M)
      G6A(M)=A1*U7+G6A(M)
      G4B(M)=A2*U5+G4B(M)
      G5B(M)=A2*U6+G5B(M)
      G6B(M)=A2*U7+G6B(M)
   30 CONTINUE
   13 CONTINUE
   47 A1=DR(IP)
      UA=A1*U3
      UB=DZ(IP)*U4
      A2=D(IP)
      D6=-A2*D2
      D7=D1*A1
      D8=D1*A2
      JM=JN
      DO 31 M=1,M3
      FM=M4+M
      A1=FM*DM(IP)
      H5A=G5A(M)
      H5B=G5B(M)
      H4A=G4A(M)+H5A
      H4B=G4B(M)+H5B
      H6A=G6A(M)
      H6B=G6B(M)
      U7=UA*H5A+UB*H4A
      U8=UA*H5B+UB*H4B
      U5=U7-U8
      U6=U7+U8
      U7=-U1*H4A
      U8=D6*H6A
      U9=D6*H6B-A1*H4A
      UC=D7*(H6A+D4*H6B)
      UD=FM*D5*H4A
      K1=IP+JM
      K2=K1+1
      K3=K1+N
```

```
      K4=K2+N
      K5=K2+MT
      K6=K4+MT
      K7=K3+N2N
      K8=K4+N2N
      GO TO (18,20,19),KQ
   18 Z(K6)=U8+U9
      IF(IP.EQ.1) GO TO 21
      Z(K3)=Z(K3)+U6-U7
      Z(K7)=Z(K7)+UC-UD
      IF(IP.EQ.MP) GO TO 22
   21 Z(K4)=U6+U7
      Z(K8)=UC+UD
      GO TO 22
   19 Z(K5)=Z(K5)+U8-U9
      IF(IP.EQ.1) GO TO 23
      Z(K1)=Z(K1)+U5+U7
      Z(K7)=Z(K7)+UC-UD
      IF(IP.EQ.MP) GO TO 22
   23 Z(K2)=Z(K2)+U5-U7
      Z(K8)=UC+UD
      GO TO 22
   20 Z(K5)=Z(K5)+U8-U9
      Z(K6)=U8+U9
      IF(IP.EQ.1) GO TO 24
      Z(K1)=Z(K1)+U5+U7
      Z(K3)=Z(K3)+U6-U7
      Z(K7)=Z(K7)+UC-UD
      IF(IP.EQ.MP) GO TO 22
   24 Z(K2)=Z(K2)+U5-U7
      Z(K4)=U6+U7
      Z(K8)=UC+UD
   22 Z(K8+MT)=U2*(D8*(H5A+D4*H5B)-A1*UD)
      JM=JM+N2
   31 CONTINUE
   16 CONTINUE
      JN=JN+N
   15 CONTINUE
      RETURN
      END
C*********************************************************SUBROUTINE SOLVE.
C REFERENCE : TECHNICAL REPORT TR-80-1
      SUBROUTINE SOLVE(N,IPS,UL,B,X)
C
C SEE MAUTZ & HARRINGTON FOR DETAILS
C
      COMPLEX UL(100000),B(800),X(800),SUM
      DIMENSION IPS(800)
      NP=N+1
      IP=IPS(1)
      X(1)=B(IP)
```

```
      DO 2 I=2,N
      IP=IPS(I)
      IPB=IP
      IM1=I-1
      SUM=(0.,0.)
      DO 1 J=1,IM1
      SUM=SUM+UL(IP)*X(J)
    1 IP=IP+N
    2 X(I)=B(IPB)-SUM
      K2=N*(N-1)
      IP=IPS(N)+K2
      X(N)=X(N)/UL(IP)
      DO 4 IBACK=2,N
      I=NP-IBACK
      K2=K2-N
      IPI=IPS(I)+K2
      IP1=I+1
      SUM=(0.,0.)
      IP=IPI
      DO 3 J=IP1,N
      IP=IP+N
    3 SUM=SUM+UL(IP)*X(J)
    4 X(I)=(X(I)-SUM)/UL(IPI)
      RETURN
      END
C***********************************************SUBROUTINE DECOMP
C REFERENCE: TECHNICAL REPORT TR-80-1
      SUBROUTINE DECOMP(N,IPS,UL)
C
C SEE MAUTZ & HARRINGTON FOR DETAILS
C
      COMPLEX UL(100000),PIVOT,EM
      DIMENSION SCL(800),IPS(800)
      DO 5 I=1,N
      IPS(I)=I
      RN=0.
      J1=I
      DO 2 J=1,N
      ULM=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))
      J1=J1+N
      IF(RN-ULM) 1,2,2
    1 RN=ULM
    2 CONTINUE
      SCL(I)=1./RN
    5 CONTINUE
      NM1=N-1
      K2=0
      DO 17 K=1,NM1
      BIG=0.
      DO 11 I=K,N
      IP=IPS(I)
```

```
      IPK=IP+K2
      SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)
      IF(SIZE-BIG) 11,11,10
   10 BIG=SIZE
      IPV=I
   11 CONTINUE
      IF(IPV-K) 14,15,14
   14 J=IPS(K)
      IPS(K)=IPS(IPV)
      IPS(IPV)=J
   15 KPP=IPS(K)+K2
      PIVOT=UL(KPP)
      KP1=K+1
      DO 16 I=KP1,N
      KP=KPP
      IP=IPS(I)+K2
      EM=-UL(IP)/PIVOT
   18 UL(IP)=-EM
      DO 16 J=KP1,N
      IP=IP+N
      KP=KP+N
      UL(IP)=UL(IP)+EM*UL(KP)
   16 CONTINUE
      K2=K2+N
   17 CONTINUE
      RETURN
      END
C**************************************************FUNCTION BLOG
C REFERENCE: TECHNICAL REPORT TR-80-1 ;(page 56)
      FUNCTION BLOG(X)
      IF(X.GT..1) GO TO 1
      X2=X*X
      BLOG=((.075*X2-.1666667)*X2+1.)*X
      RETURN
    1 BLOG=ALOG(X+SQRT(1.+X*X))
      RETURN
      END
C****************************************************SUBROUTINE PLANE
C REFERENCE: TECHNICAL REPORT TR-80-1 ;(pages 57-64)
      SUBROUTINE PLANE(M1,M2,NP,NT,RH,ZH,XT,AT,THR,R)
C
C PLANE WAVE EXCITATION VECTOR IN THE FAR FIELD. FROM MAUTZ AND HARRINGTON.
C MODIFIED TO DO ONLY ONE ANGLE AND FREQUENCY PER CALL.
C ** NEW BESSEL FUNCTION ROUTINE FROM NUM. RECIPES **
C
      COMPLEX R(1600),U,U1,UA,UB,FA(1500),FB(1500),F2A,F2B,F1A,F1B
      COMPLEX U2,U3,U4,U5
      DIMENSION RH(400),ZH(400),XT(4),AT(4),R2(4),Z2(4)
      MP=NP-1
      MT=MP-1
      N=MT+MP
```

```
      N2=2*N
      CC=COS(THR)
      SS=SIN(THR)
      U=(0.,1.)
      U1=3.141593*U**M1
      M3=M1+1
      M4=M2+3
      IF(M1.EQ.0) M3=2
      M5=M1+2
      M6=M2+2
      DO 12 IP=1,MP
      K2=IP
      I=IP+1
      DR=.5*(RH(I)-RH(IP))
      DZ=.5*(ZH(I)-ZH(IP))
      D1=SQRT(DR*DR+DZ*DZ)
      R1=.25*(RH(I)+RH(IP))
      IF(R1.EQ.0.) R1=1.
      Z1=.5*(ZH(I)+ZH(IP))
      DR=.5*DR
      D2=DR/R1
      DO 13 L=1,NT
      R2(L)=R1+DR*XT(L)
      Z2(L)=Z1+DZ*XT(L)
   13 CONTINUE
      D3=DR*CC
      D4=-DZ*SS
      D5=D1*CC
      DO 23 M=M3,M4
      FA(M)=0.
      FB(M)=0.
   23 CONTINUE
      DO 15 L=1,NT
      X=SS*R2(L)*2.
      ARG=Z2(L)*CC
      UA=AT(L)*CMPLX(COS(ARG),SIN(ARG))
      UB=XT(L)*UA
C THIS LINE REPLACES HARRINGTON'S BLOCK
      DO 25 M=M3,M4
      BES=BESSJ(M-2,X)
      FA(M)=BES*UA+FA(M)
      FB(M)=BES*UB+FB(M)
   25 CONTINUE
   15 CONTINUE
      IF(M1.NE.0) GO TO 26
      FA(1)=-FA(3)
      FB(1)=-FB(3)
   26 UA=U1
      DO 27 M=M5,M6
      M7=M-1
      M8=M+1
```

```
      F2A=UA*(FA(M8)+FA(M7))
      F2B=UA*(FB(M8)+FB(M7))
      UB=U*UA
      F1A=UB*(FA(M8)-FA(M7))
      F1B=UB*(FB(M8)-FB(M7))
      U4=D4*UA
      U2=D3*F1A+U4*FA(M)
      U3=D3*F1B+U4*FB(M)
      U4=DR*F2A
      U5=DR*F2B
      K1=K2-1
      K4=K1+N
      K5=K2+N
      R(K2+MT)=-D5*(F2A+D2*F2B)
      R(K5+MT)=D1*(F1A+D2*F1B)
      IF(IP.EQ.1) GO TO 21
      R(K1)=R(K1)+U2-U3
      R(K4)=R(K4)+U4-U5
      IF(IP.EQ.MP) GO TO 22
   21 R(K2)=U2+U3
      R(K5)=U4+U5
   22 K2=K2+N2
      UA=UB
   27 CONTINUE
   12 CONTINUE
      RETURN
      END
C*****************************************************SUBROUTINE ZLOAD.
C REFERENCE: COMPUTATION OF RADIATION AND SCATTERING FOR
C            LOADED BODIES OF REVOLUTION
C            HARRINGTON AND MAUTZ
C            REPORT AFCRL-70-0046
C            AIRFORCE CAMBRIDGE RESEARCH LABS
C            CONTRACT NO. F-19628-68-C-0180
      SUBROUTINE ZLOAD(NP,RH,ZH,ZO,Z)
C
C COMPUTES IMPEDANCE MATRIX ELEMENTS FOR LOADED BODIES OF REV
C ZO(I) IS THE SURF IMPEDANCE OF THE ITH SEGMENT (NP-1 SEGMENTS)
C Z(.) ARE THE IMPEDANCE MATRIX TERMS (TRIDIAGONAL FOR T-T
C SUBMATRIX; DIAGONAL FOR P-P SUBMATRIX). STORED IN COL VECTOR.
C
      COMPLEX C1,C2,ZO(400),Z(2400),X1,X2,X3,Y1,Y2,Y3,FN(400)
      COMPLEX U1,U2,U3,XI,YI
      DIMENSION RH(400),ZH(400),RS(400),D(400),SV(400)
      PI=3.14159
      MT=NP-2
      MP=NP-1
      N=MT+MP
      DO 10 IP=2,NP
      II=IP-1
      DR=RH(IP)-RH(II)
```

```
      DZ=ZH(IP)-ZH(II)
      D(II)=SQRT(DR*DR+DZ*DZ)
      SV(II)=DR/D(II)
      RS(II)=.5*(RH(IP)+RH(II))
      DS=D(II)*SV(II)/2.
      Q1=RS(II)+DS
      Q2=RS(II)-DS
      FN(II)=1.
      IF((ABS(Q2).GT.1.E-6).AND.(ABS(Q1).GT.1.E-6))
     * FN(II)=ALOG(Q1/Q2)
   10 CONTINUE
      LO=MT*3-2
      DO 20 I=1,MP
      C1=PI*ZO(I)
      IF(I.EQ.MP) GO TO 80
      KI=2
      IF(I.EQ.1) KI=1
      IF(I.EQ.MT) KI=3
      II=I+1
      C2=PI*ZO(II)
      A=SV(I)
      IF(ABS(A).LT.1.E-6) GO TO 41
      X1=C1*FN(I)/2./A
      X2=C1*2./A*(1.-RS(I)*FN(I)/D(I)/A)
      X3=-X2*RS(I)/D(I)/A
      GO TO 42
   41 CONTINUE
      X1=C1/2./RS(I)*D(I)
      X2=(0.,0.)
      X3=C1*D(I)/6./RS(I)
   42 CONTINUE
      A=SV(II)
      IF(ABS(A).LT.1.E-6) GO TO 45
      Y1=C2*FN(II)/2./A
      Y2=C2*2./A*(1.-RS(II)*FN(II)/D(II)/A)
      Y3=-Y2*RS(II)/D(II)/A
      GO TO 40
   45 CONTINUE
      Y1=C2/2./RS(II)*D(II)
      Y2=(0.,0.)
      Y3=C2*D(II)/6./RS(II)
   40 CONTINUE
C
C DEFINE TRIDIAGONAL ELEMENTS FOR T-T SUBMATRIX (STORED IN COLS)
C (U1- DIAG; U2- LOWER; U3- UPPER)
C
      XI=X1+X2+X3
      YI=Y1-Y2+Y3
      IF(RH(I).LT.1.E-6) XI=C1/SV(I)
      IF(RH(II).LT.1.E-6) YI=C2/SV(II)
      U1=XI+YI
```

```
      U2=X1-X3
      U3=Y1-Y3
      L=2+(I-2)*3
      IF(KI.EQ.1) L=0
      L1=L+1
      L2=L+2
      L3=L+3
      go to (50,60,70),ki
   50 Z(L1)=U1
      Z(L2)=U2
      GO TO 80
   60 Z(L1)=U3
      Z(L2)=U1
      Z(L3)=U2
      GO TO 80
   70 Z(L1)=U3
      Z(L2)=U1
   80 Z(L0+I)=2.*C1*D(I)/RS(I)
   20 CONTINUE
      RETURN
      END
      SUBROUTINE ZTOT(MT,MP,ZL,Z)
C
C ADDS THE SURF IMPEDANCE TERMS TO THE TRIDIAGONAL ELEMENTS OF
C THE BOR IMPEDANCE MATRIX Z.
C
      COMPLEX ZL(2400),Z(100000)
      N=MT+MP
      M0=MT*3-2
      DO 100 I=1,MP
      L0=MT*N+(I-1)*N+MT
      IF(I.EQ.MP) GO TO 80
      KI=2
      IF(I.EQ.1) KI=1
      IF(I.EQ.MT) KI=3
      L2=(I-1)*N+I
      L1=L2-1
      L3=L2+1
      M=2+3*(I-2)
      IF(KI.EQ.1) M=0
      M1=M+1
      M2=M+2
      M3=M+3
      go to (50,60,70),ki
   50 Z(L2)=Z(L2)+ZL(M1)
      Z(L3)=Z(L3)+ZL(M2)
      GO TO 80
   60 Z(L1)=Z(L1)+ZL(M1)
      Z(L2)=Z(L2)+ZL(M2)
      Z(L3)=Z(L3)+ZL(M3)
      GO TO 80
```

```
   70 Z(L1)=Z(L1)+ZL(M1)
      Z(L2)=Z(L2)+ZL(M2)
   80 Z(L0+I)=Z(L0+I)+ZL(M0+I)
  100 CONTINUE
      RETURN
      END
C*************************************************SUBROUTINE OGIVE
C SUBROUTINE  : OGIVE
C DATE        : 4 SEPTEMBER 1991
C PROGRAMMER  : R.M. FRANCIS
C REVISED     : 26 JANUARY 1992
C COMMENTS    :
C THIS SUBROUTINE WILL GENERATE DATA FOR A BODY OF REVOLUTION (BOR) IN
C THE FORM OF AN OGIVE.
C DIMENSIONS ARE NORMALIZED TO WAVELENGTH, SEE PAGE 14 FRANCIS THESIS.
C ZH = Z CO-ORDINATE * 2*PI
C RH = RADIUS *2*PI
      SUBROUTINE  OGIVE(NP,ARAD,ZH,RH,BASE,RS,ZP)
C NP = NUMBER OF POINTS ON THE OGIVE SURFACE, MAXIMUM = 1000
C ZP   = ZPRIME, THE POSITION ON Z WHERE THE RADIUS OF CURVATURE STARTS
C BASE = BASE RADIUS
C RS   = RADIUS OF CURVATURE IN THE RZ,Z PLANE OF THE OGIVE
      INTEGER I,NP,NPBASE
      REAL  RH(400),ZH(400),ZP,BASE,RS,ZCOORD,RADIUS,AL
      PI=3.1415926
C INPUT THE VARIABLES FOR THE OGIVE,ZP,B,RS,NP
      WRITE(6,*) 'ENTER SURFACE CURVATURE (wavelengths)'
      READ(5,*) RS     .                    .
      WRITE(6,*) 'ENTER ZPRIME,WHERE CURVATURE STARTS (wavelengths)'
      READ(5,*) ZP
      WRITE(6,*) 'ENTER BASE RADIUS (wavelengths)'
      READ(5,*) BASE
C PERFORM CALCULATIONS
           AL=SQRT(2.*BASE*RS-BASE**2)
         ZMAX= AL + ZP
          ANG=ASIN(AL/RS)
            L=ANINT((RS*ANG+ZP)*5)
           NP=2*L+1

         WRITE(6,*)'NUMBER OF POINTS IS: ',NP
         DZ= ZMAX/FLOAT(NP-1)
      DO 10  I=1,NP
         ZCOORD=ZMAX- FLOAT(I-1)*DZ
           IF(ZCOORD.EQ.0.)THEN
               ZCOORD=0.0000000001
           ENDIF
         ZH(I)=2.*PI*ZCOORD
           IF (ZCOORD.LE.ZP) THEN
               RADIUS=BASE
           ELSE
              RADIUS=SQRT(RS**2-(ZCOORD-ZP)**2)+(BASE-RS)
```

```
            IF(RADIUS.EQ.0.)THEN
               RADIUS=0.0000000001
            ENDIF
         ENDIF
        RH(I)=2.*PI*RADIUS
   10 CONTINUE
      RETURN
      END
C*****************************************************SUBROUTINE GENEX
C SUBROUTINE  : GENEX
C DATE        : 14 JANUARY 1992
C REVISED     : 17 February 1992
C PROGRAMMER  : R.M. FRANCIS
      SUBROUTINE GENEX(MODE,NP,NT,NPHI,CNPHI,XT,AT,X,A,
     *                 XP,AP,THS,PHS,ARAD,RHO,ZHO,R)
C SOURCE ON Z AXIS AT Z=0. THIS VERSION DOES THE ANTENNA INTEGRATION
C IN RECTANGULAR COORDINATES.  CNPHI POINTS USED IN X AND Y.
C ****************************
C                            *
C      t          t    i     *
C   ( V )    = < W   , E >   *
C      n  i       ni         *
C                            *
C                            *
C      phi        phi  i     *
C   ( V )    = < W   , E >   *
C      n  i       ni         *
C                            *
C****************************
C THE EXCITATION VECTOR IS COMPUTED FOR THE GIVEN R,THETA AND PHI
C COMPONENTS ON SURFACE OF BOR, SPECIFIED IN SUBROUTINE CIRCRTP.
      COMPLEX R(800),CEXP,PSI,S1,S2,S3,S4,S5
      COMPLEX CIRCR,CIRCT,CIRCP
      DIMENSION RH(400),ZH(400),XT(4),AT(4),X(100),A(100)
      DIMENSION XP(100),AP(100),RHO(400),ZHO(400)
      INTEGER CNPHI
      PI=3.141592654
      BK=2.*PI
      MP=NP-1
      MT=MP-1
      N=MT+MP
C LIMITS ON PHI INTEGRATION
      P1=(2.*PI-0.)/2.
      P2=(2.*PI+0.)/2.
      DO 10 I=1,NP
      RH(I)=RHO(I)/BK
10    ZH(I)=ZHO(I)/BK
      DO 30 IP=1,MP
C QUANTITIES FOR THE FIRST SEGMENT (POSITIVE SLOPE)
      I=IP+1
      II=IP
```

```
      DR=RH(I)-RH(II)
      DZ=ZH(I)-ZH(II)
      D1=SQRT(DR*DR+DZ*DZ)
      R1=.5*(RH(I)+RH(II))
      Z1=.5*(ZH(I)+ZH(II))
      SVP1=DR/D1
      CVP1=DZ/D1
      V1=ATAN2(DR,DZ+1.E-5)
C QUANTITIES FOR THE SECOND SEGMENT (NEGATIVE SLOPE)
C (SKIP IF LAST SEGMENT)
      I=IP+2
      II=IP+1
      DR=RH(I)-RH(II)
      DZ=ZH(I)-ZH(II)
      D2=SQRT(DR*DR+DZ*DZ)
      R2=.5*(RH(I)+RH(II))
      Z2=.5*(ZH(I)+ZH(II))
      SVP2=DR/D2
      CVP2=DZ/D2
      V2=ATAN2(DR,DZ+1.E-5)
C BEGIN PHI INTEGRATION: R TERMS IN S1 AND S2; THETA TERM IN S3 AND S4;
C PHI TERM IN S5.
      S1=(0.,0.)
      S2=(0.,0.)
      S3=(0.,0.)
      S4=(0.,0.)
      S5=(0.,0.)
      DO 20 J=1,NPHI   .
      PH=P1*X(J)+P2
      PSI=CEXP(CMPLX(0.,-MODE*PH))*A(J)
      IF(IP.LE.MT) THEN
C t-CURRENT INTEGRATION FOR THE POSITIVE SLOPE
C Gauss Quadrature
      DO 13 L=1,NT
      TP=D1*XT(L)/2.
      RHB=R1+TP*SVP1
      ZHB=Z1+TP*CVP1
      TH=ATAN2(RHB,ZHB+1.E-5)
      CC=COS(V1-TH)
      SS=SIN(V1-TH)
      CALL CIRCRTP(CNPHI,XP,AP,ARAD,THS,PHS,
     *                     PH,RHB,ZHB,CIRCR,CIRCT,CIRCP)
      S1=S1+AT(L)*(.5+TP/D1)*CC*CIRCR*PSI
      S2=S2+AT(L)*(.5+TP/D1)*SS*CIRCT*PSI
13    CONTINUE
C t-CURRENT INTEGRATION FOR THE NEGATIVE SLOPE
C Gauss Quadrature
      DO 14 L=1,NT
      TP=D2*XT(L)/2.
      RHB=R2+TP*SVP2
      ZHB=Z2+TP*CVP2
```

```
      TH=ATAN2(RHB,ZHB+1.E-5)
      SS=SIN(V2-TH)
      CC=COS(V2-TH)
      CALL CIRCRTP(CNPHI,XP,AP,ARAD,THS,PHS,
     *                      PH,RHB,ZHB,CIRCR,CIRCT,CIRCP)
      S3=S3+AT(L)*(.5-TP/D2)*CC*CIRCR*PSI
      S4=S4+AT(L)*(.5-TP/D2)*SS*CIRCT*PSI
14    CONTINUE
      ENDIF
C phi-CURRENT INTEGRATION
      DO 15 L=1,NT
      TP=D1*XT(L)/2.
      RHB=R1+TP*SVP1
      ZHB=Z1+TP*CVP1
      TH=ATAN2(RHB,ZHB+1.E-5)
      CALL CIRCRTP(CNPHI,XP,AP,ARAD,THS,PHS,
     *                      PH,RHB,ZHB,CIRCR,CIRCT,CIRCP)
      S5=S5+AT(L)*CIRCP*RHB/R1*PSI
15    CONTINUE
20    CONTINUE
C COMPONENTS ARE STORED IN A COLUMN VECTOR: VT(1,MT), VP(MT+1,N)
      IF(IP.LE.MT) THEN
      R(IP)=(S1+S2)*D1/2.*P1+(S3+S4)*D2/2.*P1
      ENDIF
      R(IP+MT)=S5*P1*D1/2.
30    CONTINUE
      RETURN
      END
C***************************************************SUBROUTINE CIRCRTP
C SUBROUTINE   :        CIRCRTP
C DATE         :        1 OCTOBER 1991
C REVISED      :        26 February 1992
C PROGRAMMER   :        R.M. FRANCIS
C 2ND REVISION :        27 October 1992
C PROGRAMMER   :        K. A. KLOPP
C
C COMMENTS :  THIS SUBROUTINE WILL RIGOROUSLY CALCULATE THE ELECTRIC FIELD
C FOR A CIRCULAR APERTURE. THE FIELD IS CALCULATED AT THE COORDINATES
C SPECIFIED BY RH(.) AND ZH(.)  THE APERTURE IS LOCATED AT Z = 0,
C AND SCANNED TO A DIRECTION (THS,PHS). THE SUBROUTINE OGIVE IS THE
C SOURCE OF THE GEOMETRIC DATA REQUIRED BY CIRC TO PERFORM COMPUTATIONS.
C ALL PHYSICAL DIMENSIONS ARE NORMALIZED TO WAVELENGTH.
C
C*****************************************************************************
C                      +ant   +ant                                          *
C                        _      _                                           *
C    i                   \      \     x1                                    *
C   E  = -jn/(4*pi*k)   /_     /_    e (.......)*TAPER(rp)                  *
C                       x=0    y=0                                          *
C                                                                           *
C*****************************************************************************
```

```
      SUBROUTINE CIRCRTP(CNPHI,XP,AP,ARAD,THS,PHS,
     *                           PHI,RZ,Z,CIRCR,CIRCT,CIRCP)
      INTEGER CNPHI
      DIMENSION XP(100),AP(100)
      REAL K
      COMPLEX J,JK,G1,G2,X1,CON,CC,SUMX,SUMY,SUMZ
      COMPLEX CIRCR,CIRCT,CIRCP
      PI=3.141592654
      ETA=120.*PI
C SET FIELD COMPONENTS TO ZERO IN THE REAR HEMISPHERE
      CIRCR=(0.,0.)
      CIRCT=(0.,0.)
      CIRCP=(0.,0.)
C Z OF THE OBSERVATION POINT IS >= 0 IN THE FORWARD HEMISPHERE
      IF(Z.GE.0.) THEN
      SUMX=(0.0,0.0)
      SUMY=(0.0,0.0)
      SUMZ=(0.0,0.0)
      K=6.283185307
      J = (0.0,1.0)
      JK= (0.0,6.283185307)
      CON=-J/(4.*PI)
C OMIT THE FACTOR OF ETA SINCE IT IS NOT INCLUDED IN SUBROUTINE ZMAT.
C ARAD**2 IS FOR SCALING OF GAUSSIAN INTEGRATION.  READJUST SCALING
C TO AGREE WITH OLD SUBROUTINE SPHERE (ARBITRARY)
      CC= CON
C OUTER  LOOP: INTEGRATE OVER X, -ARAD < X < ARAD.
C INNER  LOOP: INTEGRATE OVER Y, -ARAD < Y < ARAD.
C EVALUATE ANTENNA FIELD AT RZ,Z:
      XO=RZ*COS(PHI)
      YO=RZ*SIN(PHI)
        STS=SIN(-THS)
        S=RZ/SQRT(RZ**2+Z**2)
        C=Z/SQRT(RZ**2+Z**2)
C INTEGRATE IN X,Y COORDINATES
          DO 50 M=1,CNPHI
             X=ARAD*XP(M)
            DO 60 N=1,CNPHI
             Y=ARAD*XP(N)
       RP=SQRT(X**2+Y**2)
C SKIP INTEGRATION POINTS BEYOND THE ANTENNA RADIUS
      IF(RP.LE.ARAD) THEN
      PHIPRIME=ATAN2(Y,X+1.E-10)
      R=SQRT(((RZ-RP)**2+Z**2)+(4*RZ*RP*SIN((PHI-PHIPRIME)/2)**2))
      G1=(((K*R)**2)-1-(JK*R))/R**3
      G2=(3+(3*JK*R)-(K*R)**2)/R**5
      X1=JK*(RP*COS(PHIPRIME-PHS)*STS-R)
      Y1=XO-X
      X2=Y1**2
      Y2=YO-Y
         SUMX=SUMX+(CC*AP(M)*AP(N)*(G1+(X2*G2))*CEXP(X1))*TAPER(RP)
```

```
         SUMY=SUMY+(CC*AP(M)*AP(N)*Y1*Y2*G2*CEXP(X1))*TAPER(RP)
         SUMZ=SUMZ+(CC*AP(M)*AP(N)*Y1*Z*G2*CEXP(X1))*TAPER(RP)
      ENDIF
   60    CONTINUE
   50    CONTINUE
C ASSUME AN ELEMENT FACTOR, ELMT
c     ELMT=SQRT(ABS(C))
      ELMT=1.
      CIRCT=(C*COS(PHI)*SUMX+C*SIN(PHI)*SUMY-S*SUMZ)*ELMT
      CIRCR=(S*COS(PHI)*SUMX+S*SIN(PHI)*SUMY+C*SUMZ)*ELMT
      CIRCP=(-SIN(PHI)*SUMX+COS(PHI)*SUMY)*ELMT
      ENDIF
      RETURN
      END
C***********************************************SUBROUTINE TESTSPHERE
C SUBROUTINE     :   TESTSPHERE
C DATE           :   21 FEBRUARY 1992
C PROGRAMMER     :   R. M. FRANCIS
C COMMENTS       :   GENERATES A SPHERE, WITH PLOTTED POINTS
C                    AT EQUAL INTERVALS IN THETA.
      SUBROUTINE TESTSPHERE(NP,ZH,RH,BASE,RS,ZP)
      INTEGER NP,I
      REAL ZH(400),RH(400),BASE,RS,ZP,SPRAD
      PI=3.1415926
      BK=2.*PI
      ZP=0.0
      WRITE(6,*)'ENTER NUMBER OF POINTS (NP):'
      READ(5,*)NP
      WRITE(6,*)'ENTER SPHERE RADIUS'
      READ(5,*)SPRAD
      BASE=SPRAD
        RS=SPRAD
      DO 1241 I=1,NP
         ANGLE=PI*FLOAT(I-1)/(2.*FLOAT(NP-1))
         ZH(I)=BK*SPRAD*COS(ANGLE)
         RH(I)=BK*SPRAD*SIN(ANGLE)
           IF(RH(I).EQ.0.)THEN
               RH(I)=0.0000000001
           ENDIF
1241  CONTINUE
      RETURN
      END
C***************************************************SUBROUTINE CONE
C GENERATES THE BOR GEOMETRY FOR A CONE.
      SUBROUTINE CONE(NP,ZH,RH,BASE,RS,ZP)
      REAL ZH(400),RH(400),BASE,RS,ZP
      REAL HA,ANG,ZMAX,DZH
      INTEGER NP
      PI=3.1415926
      BK=2.*PI
      WRITE(6,*)'ENTER THE CONE HALF ANGLE IN DEGREES'
```

```
      READ(5,*)HA
         IF(HA.GT.90.)THEN
         WRITE(6,*)'RANGE OF ANGLE: 0< ANGLE<90'
         WRITE(6,*)'RANGE EXCEEDED'
         GOTO 10
         ENDIF
      ANG=HA*PI/180.
      WRITE(6,*)'ENTER THE MAXIMUM Z'
      READ(5,*)ZMAX
      WRITE(6,*)'ENTER THE NUMBER OF POINTS (NP)'
      READ(5,*)NP
      ZH(1)=ZMAX*BK
      RH(1)=0.
      DZH=ZH(1)/FLOAT(NP-1.)
      DO 10 I=1,NP
         IF(ZH(I).EQ.0.)THEN
            ZH(I)=.00000001
         ENDIF
      ZH(I+1)=ZH(I)-DZH
      RH(I+1)=RH(I)+(DZH*SIN(ANG))
   10 CONTINUE
      BASE=RH(NP)/BK
      ZP=0.
      RS=0.
      RETURN
      END
C***********************************************SUBROUTINE DISK
       SUBROUTINE DISK(NP,ZH,RH,BASE,RS,ZP)
      REAL ZH(400),RH(400),BASE,RS,ZP,RMAX
      INTEGER NP
      WRITE(6,*)'ENTER THE RADIUS OF THE DISK'
      READ(5,*)RMAX
      WRITE(6,*)'ENTER THE DISTANCE TO THE DISK'
      READ(5,*)ZP
      WRITE(6,*)'ENTER THE NUMBER OF POINTS (NP)'
      READ(5,*)NP
      PI=3.1415926
      BK=2.*PI
      DRH=RMAX*BK/FLOAT(NP-1.)
      DO 10 I=1,NP
         ZH(I)=ZP*BK
         RH(I)=DRH*FLOAT(I-1.)
   10 CONTINUE
      BASE=RMAX
      RS=0.
      RETURN
      END
C***********************************************SUBROUTINE PARAB
      SUBROUTINE PARAB(NP,ZH,RH,DM,FOD,ZP)
      DIMENSION RH(400),ZH(400)
      WRITE(6,*) 'ENTER DIAMETER AND F/D RATIO'
```

```
      READ(5,*) DM,FOD
      WRITE(6,*) 'FEED DISTANCE FROM FOCUS (+ IS NEARER)'
      READ(5,*) ZP
      WRITE(6,*) 'ENTER NUMBER OF GENERATING POINTS'
      READ(5,*) NP
      BK=2.*3.14159
      FM=FOD*DM
      PHIV=2.*ATAN(1./4./FOD)
      DO 24 I=1,NP
      TH=FLOAT(I-1)*PHIV/FLOAT(NP-1)
      RM=2.*FM/(1.+COS(TH))*BK
      ZH(I)=RM*COS(TH)+ZP*BK
      RH(I)=RM*SIN(TH)
24    CONTINUE
      RETURN
      END
C*******************************************FUNCTION TAPER
      FUNCTION TAPER(RHO)
C SPECIFY ANTENNA ILLUMINATION FUNCTION.  REAL FUNCTION OF
C RHO ONLY
      TAPER=1.
      RETURN
      END
C******************************************SUBROUTINE ANTFF
      SUBROUTINE ANTFF(THETA,PHI,THETAS,PHIS,ARAD,ETF,EPF)
C COMPUTES CLOSED FORM ANTENNA PATTERN IN THE FAR FIELD.
C ETF AND EPF ARE RETURNED. ANGLES ARE IN RADIANS.
C PRESENT ENTRY IS FOR:
C
C UNIFORM ILLUMINATION SCANNED TO A DIRECTION (THETAS,PHIS)
C
      COMPLEX ETF,EPF,JK,SCL,EB
      JK=(0.0,6.283185307)
      BK=6.283185307
      PI=BK/2.
C SET RADIATION PATTERN TO ZERO IN THE REAR HEMISPHERE
      IF(COS(THETA).LT.0.) THEN
        EB=(0.,0.)
      ELSE
        BB=SIN(THETA)*SIN(PHI)-SIN(THETAS)*SIN(PHIS)
        AA=SIN(THETA)*COS(PHI)-SIN(THETAS)*COS(PHIS)
        ARG=BK*ARAD*SQRT(AA**2+BB**2)
        SCL=-(0.,1.)*2.*PI**2
        IF(ABS(ARG).LT.1.E-5)THEN
          EB=SCL/2.
          ELSE
          EB=SCL*(BESSJ1(ARG)/ARG)
        ENDIF
      ENDIF
C EB IS THE X COMPONENT; GET ETHETA (ETF) AND EPHI (EPF)
C ASSUME AN ELEMENT FACTOR, ELMT
```

```
       ELMT=SQRT(ABS(COS(THETA)))
      ELMT=1.
       ETF=EB*COS(THETA)*COS(PHI)*ELMT
       EPF=-EB*SIN(PHI)*ELMT
      RETURN
      END
      FUNCTION BESSJ(N,X)
C RETURNS THE BESSEL FUNCTION B OF ORDER N (>1) AND REAL
C ARGUMENT X.
      PARAMETER (IACC=40,BIGNO=1.E10,BIGNI=1.E-10)
      IF(N.EQ.0) THEN
C IF N=0 CALL BESSJ0
        BESSJ=BESSJ0(X)
       ELSE IF(N.EQ.1) THEN
C IF N=1 CALL BESSJ1
        BESSJ=BESSJ1(X)
       ELSE
C IF N>1 USE RECURSION
      BESSJ=0.
      IF(X.NE.0.) THEN
      TOX=2./X
      IF(X.GT.FLOAT(N)) THEN
      BJM=BESSJ0(X)
      BJ=BESSJ1(X)
      DO 11 J=1,N-1
      BJP=J*TOX*BJ-BJM
      BJM=BJ
      BJ=BJP
   11 CONTINUE
      BESSJ=BJ
      ELSE
      M=2*((N+INT(SQRT(FLOAT(IACC*N))))/2)
      BESSJ=0.
      JSUM=0.
      SUM=0.
      BJP=0.
      BJ=1.
      DO 12 J=M,1,-1
      BJM=J*TOX*BJ-BJP
      BJP=BJ
      BJ=BJM
      IF(ABS(BJ).GT.BIGNO) THEN
      BJ=BJ*BIGNI
      BJP=BJP*BIGNI
      BESSJ=BESSJ*BIGNI
      SUM=SUM*BIGNI
      ENDIF
      IF(JSUM.NE.0) SUM=SUM+BJ
      JSUM=1-JSUM
      IF(J.EQ.N) BESSJ=BJP
   12 CONTINUE
```

```
      SUM=2.*SUM-BJ
      BESSJ=BESSJ/SUM
      ENDIF
      ENDIF
      ENDIF
      RETURN
      END
      FUNCTION BESSJ0(X)
C
C BESSEL FUNCTION OF 0 ORDER, REAL ARGUMENT X
C (SEE 'NUMERICAL RECIPES', P.172)
C
      REAL*8 Y,P1,P2,P3,P4,P5,Q1,Q2,Q3,Q4,Q5,R1,R2,R3,R4,R5,R6,
     * S1,S2,S3,S4,S5,S6
      DATA P1,P2,P3,P4,P5/1.D0,-.109862827D-2,.2734510407D-4,
     * -.2073370639D-5,.2093887211D-6/
      DATA Q1,Q2,Q3,Q4,Q5/-.1562499995D-1,.1430488765D-3,
     * -.6911147651D-5,.7621095161D-6,-.934945152D-7/
      DATA R1,R2,R3,R4,R5,R6/57568490574.D0,-13362590354.D0,
     * 651619640.7D0,-11214424.18D0,77392.33017D0,-184.9052456D0/
      DATA S1,S2,S3,S4,S5,S6/57568490411.D0,1029532985.D0,
     * 9494680.718D0,59272.64853D0,267.8532712D0,1.D0/
      IF(X.EQ.0.) THEN
       BESSJ0=1.
      ELSE IF(ABS(X).LT.8.) THEN
      Y=X**2
      BESSJ0=(R1+Y*(R2+Y*(R3+Y*(R4+Y*(R5+Y*R6)))))/
     *    (S1+Y*(S2+Y*(S3+Y*(S4+Y*(S5+Y*S6)))))
      ELSE
      AX=ABS(X)
      Z=8./AX
      Y=Z**2
      XX=AX-.785398164
      BESSJ0=SQRT(.636619772/AX)*(COS(XX)*(P1+Y*(P2+Y*(P3+
     *    Y*(P4+Y*P5))))-Z*SIN(XX)*(Q1+Y*(Q2+Y*(Q3+
     *    Y*(Q4+Y*Q5)))))
      ENDIF
      RETURN
      END
      FUNCTION BESSJ1(X)
C
C BESSEL FUNCTION B OF ORDER 1, REAL ARGUMENT X
C (SEE 'NUMERICAL RECIPES', P.173)
C
      REAL*8 Y,P1,P2,P3,P4,P5,Q1,Q2,Q3,Q4,Q5,R1,R2,R3,R4,R5,R6,
     * S1,S2,S3,S4,S5,S6
      DATA P1,P2,P3,P4,P5/1.D0,.183105D-2,-.3516396496D-4,
     * .2457520174D-5,-.20337019D-6/
      DATA Q1,Q2,Q3,Q4,Q5/.04687499995D0,-.2002690873D-3,
     * .8449199096D-5,-.99228987D-6,.105787412D-6/
      DATA R1,R2,R3,R4,R5,R6/72362614232.D0,-7895059235.D0,
```

```
* 242396853.1D0,-2972611.439D0,15704.4826D0,-30.16036606D0/
 DATA S1,S2,S3,S4,S5,S6/144725228442.D0,2300535178.D0,
* 18583304.74D0,99447.43394D0,376.9991397D0,1.D0/
 IF(X.EQ.0.) THEN
  BESSJ1=0.
 ELSE IF(ABS(X).LT.8.) THEN
 Y=X**2
 BESSJ1=X*(R1+Y*(R2+Y*(R3+Y*(R4+Y*(R5+Y*R6)))))/
*    (S1+Y*(S2+Y*(S3+Y*(S4+Y*(S5+Y*S6)))))
 ELSE
 AX=ABS(X)
 Z=8./AX
 Y=Z**2
 XX=AX-2.356194491
 BESSJ1=SQRT(.636619772/AX)*(COS(XX)*(P1+Y*(P2+Y*(P3+
*    Y*(P4+Y*P5))))-Z*SIN(XX)*(Q1+Y*(Q2+Y*(Q3+
*    Y*(Q4+Y*Q5)))))*SIGN(1.,X)
 ENDIF
 RETURN
 END
```

APPENDIX C. GAIN.F LISTING

Material on the following pages is a listing of the program *GAIN.F* described in chapter III.

```
C*******************************************************************
C
C  PROGRAM     : gain.f
C  DATE        : 17 November 1992
C  PROGRAMMERS : Dr. D. Jenn, LT K. Klopp
C
C  Computes the gain of a circular aperature antenna radiating through
C  a radome consisting of a body of revolution, by numerically
C  integrating electric field surrounding system.  E-Field is generated by
C  currents distributed on a surface of the body of revolution.  The
C  currents, and their locations were previously determined using the
C  program "radome.f", and stored in the file "f77curcoefsA or B."
C
C  INPUT < f77curcoefsA/B
C
C*******************************************************************
      CHARACTER ITH
      CHARACTER*12 CC
      CHARACTER*10 GO
      CHARACTER*8 GNT,GNPHI,CGP
      CHARACTER*14 TPTS,PPTS,PHIPTS
      COMPLEX R(1600),LEADTERM,ETT,EPT
      COMPLEX EP,ET,ETF,EPF,ETCOMB,EPCOMB
      COMPLEX EXP1,EXP2,CONJG,CEXP,CMPLX,CT(30000),IMP,JK
      COMPLEX U,UC,ET1,EP1,ET2,EP2,ZL0(400)
C      DIMENSION RH(400),ZH(400),IPS(800)
      DIMENSION RH(400),ZH(400)
      DIMENSION XT(4),AT(4),A(100),X(100),XP(100),AP(100)
      DIMENSION Q(30),S(30)
      DIMENSION THRR(1000,1000),PHRR(1000)
C      DIMENSION ETSCAT(500),EPSCAT(500)
C      DIMENSION EXP(500),ANG(500),ECP(500)
      INTEGER CNPHI,SELECTION
      DATA PI/3.1415926/
      DATA RR/1000/
      Rad=PI/180.
      BK=2.*PI
      U=(0.,1.)
      U0=(0.,0.)
      UC=-U/4./PI
      JK=(0.0,6.283185307)
C*****ENTER A LETTER TO INDICATE WHICH ITERATION THIS IS*****************
C*****   (ALL DATA FILES ARE APPENDED WITH THIS LETTER)   ***********
C*****   (.m FILES ARE FOR GENERATING PLOTS IN MATLAB)    ***********
      WRITE(6,*)'ENTER A LETTER TO INDICATE WHICH ITERATION THIS IS'
      READ(5,*)ITH
      CC='curcoefsdat'//ITH
      GO='gainout'//ITH//'.m'
C*******************************************************************
C
C  READ in the following data from file called f77curcoefs:
```

```
C
C        iteration,                  ITH
C        geometry of BOR,            SELECTION
C        radius of base of BOR,      BASE
C        total points,               NP
C        number of t-intervals,      MT
C        phi-intervals,              MP
C        sum of t and phi intervals, N
C        number of rows of currents, NROW
C        total number of modes,      MODES
C        number of sub vectors,      NBLOCK
C        antenna scan angle in theta, THS
C        antenna scan angle in phi,  PHS
C        antenna radius,             ARAD
C        observation angle,          PHIO
C        complex radome impedance,   IMP
C        zprime,                     ZP
C        surface radius,             RS
C        closed form antenna pattern, IFF=0
C
C        files where Gauss integration weights and abscissas are stored
C
C        coordinates of surface points *2PI/lamda, ZH(I), and RH(I)
C
C************************************************************************
      OPEN(1,file=CC)
      WRITE(6,*) 'FILE OPENED IS ',CC
      READ(1,*) ITH,SELECTION,BASE,NP,MT,MP,N
      READ(1,*) NROW,MODES,NBLOCK,THS,PHS,ARAD,PHIO
      READ(1,*) IMP,ZP,RS,IFF
c      write(6,*)IMP,ZP,RS,IFF
      READ(1,*) (RH(L),L=1,NP)
c      write(6,*) (RH(L),L=1,NP)
      READ(1,*) (ZH(L),L=1,NP)
c      write(6,*) (ZH(L),L=1,NP)
      READ(1,*) (ZLO(L),L=1,NP)
C      write(6,*) (ZLO(L),L=1,NP)
C*************************************************************************
C
C  READ in current coefficients, CT(I) from file curcoefsdatA/B
C  (This is one one collumn vector.)
C
C*************************************************************************
      READ(1,*) (CT(L),L=1,NROW)
C       write(6,*)(CT(L),L=1,NROW)
      READ(1,*) GNT,GNPHI,CGP
      WRITE(6,*) 'RUN CODE READ FROM DISC : ',ITH
      THETAS=THS*RAD
      PHIS=PHS*RAD
      MHI=MODES+1
      CLOSE(1)
```

```
C***************************************************************************
C
C   Read in weights A(I), and Abscissas X(I) for the following Gauss-
C   Quadrature Integrations:
C                                                           file      wts  absc
C
C      BOR integration in t direction:         TPTS   = gaus/gaus#; XT(I),AT(I)
C      BOR integration in phi direction:       NPHI   = gaus/gaus#; X (I),A (I)
C      Antenna integration in x,y direction:   PHIPTS = gaus/gaus#; XP(I),AP(I)
C
C***************************************************************************
      TPTS='gaus/'//GNT
      PPTS='gaus/'//GNPHI
      PHIPTS='gaus/'//CGP
      OPEN(2,FILE=TPTS,STATUS='OLD')
      OPEN(3,FILE=PPTS,STATUS='OLD')
      OPEN(8,FILE=PHIPTS,STATUS='OLD')
      READ(2,*)NT
      WRITE(6,*)'NT = ',NT
         IF(NT.GT.4)THEN
           WRITE(6,*)'MAXIMUM NUMBER OF POINTS(NT) IS 4'
           GOTO 999
         ENDIF
      READ(3,*)NPHI
      WRITE(6,*)'NPHI = ',NPHI
         IF(NPHI.GT.200)THEN
           WRITE(6,*)'MAXIMUM NUMBER OF POINTS(NPHI) IS 200'
           GOTO 999
         ENDIF
      READ(8,*)CNPHI
      WRITE(6,*)'CNPHI = ',CNPHI
         IF(CNPHI.GT.100)THEN
           WRITE(6,*)'MAXIMUM NUMBER OF POINTS(CNPHI) IS 100'
           GOTO 999
         ENDIF
C LOAD THE WEIGHTS AND ABSCISSAS IN THE VECTORS.
      DO 1 M=1,NT
         READ(2,*,END=1)XT(M),AT(M)
         WRITE(6,*)XT(M),AT(M)
  1   CONTINUE
      DO 2 M=1,NPHI
          READ(3,*,END=2)X(M),A(M)
          WRITE(6,*)X(M),A(M)
  2   CONTINUE
      DO 4 M=1,CNPHI
          READ(8,*,END=4)XP(M),AP(M)
          WRITE(6,*)XP(M),AP(M)
  4   CONTINUE
      CLOSE(2)
      CLOSE(3)
      CLOSE(8)
```

```
C*******************************************************************************
C
C   Create an output file called 'gainoutA/B.m',and output data on geometry etc.
C
C*******************************************************************************
      OPEN(7,FILE=GO)
      WRITE(7,2000)' GEOMETRY = ',SELECTION,' NP = ',NP
     &,           ' MT = ',MT,' MP = ',MP,' N = ',N
     &,           ' MODES = ',MODES,' NBLOCK = ',NBLOCK
     &,           ' PHI = ',PHI
      WRITE(7,2001)' BASE = ',BASE,' ARAD = ',ARAD,' THETASCAN = ',THS
     &,           ' PHISCAN = ',PHS
      WRITE(7,*)'NPHI =',NPHI
2000  FORMAT(//5X,'%% RADOME GAIN CALCULATION %%'//,10(/'% ',A13,I4))
2001  FORMAT(3(/'% ',A13,F9.2),/)
C*******************************************************************************
C
C   Gain Computation:
C                              4 pi *  Max Radiation Intensity
C                        G = ____________________________________________
C                            Integration Over Closed Surface ( Radiation
C                            Intensity * Beam Solid Angle )
C                                                        _
C   Radiation Intensity = (1/(2*eta))*| E(t,phi,r) |**2
C          _
C   Since  E is in the far field, need to only determine Etheta, Ephi.
C    _
C  |E|**2 = |E scattered|**2 + |E antenna|**2
C
C  |E scattered| = Etheta + Ephi
C
C*******************************************************************************
C**Enter # divisions for integration in theta, and phi
C  directions. (This allows easy variation of the number of integration steps
C  without having to recompute a different set of weights and abscissas for
C  Gauss Integration.)
      WRITE(6,*)'ENTER NUMBER OF DIVISIONS IN THETA DIRECTION',
     &          ' NDIVT = ?'
      READ(5,*)NDIVT
      WRITE(6,*)'NDIVT = ',NDIVT
      WRITE(7,*)'% NDIVT = ',NDIVT
      WRITE(6,*)'ENTER NUMBER OF DIVISIONS IN PHI DIRECTION, NDIVP = ?'
      READ(5,*)NDIVP
      WRITE(6,*)'NDIVP = ',NDIVP
      WRITE(7,*)'% NDIVP = ',NDIVP
      kk=NDIVT
C      do 500 NDIVT=1,kk
       NDIVP=NDIVT
C************************************integration intervals 0<theta<180, 0<phi<360
      THETA1=0
      THETA2=180
```

```
      S1=THETA1
      S2=THETA2
      S1=S1*Rad
      S2=S2*Rad
      PHI1=0
      PHI2=360
      Q1=PHI1
      Q2=PHI2
      Q1=Q1*Rad
      Q2=Q2*Rad
C************************************theta integration angles
      DS=(S2-S1)/FLOAT(NDIVT)
      DO 200 I=1,NDIVT+1
        S(I)=(I-1)*DS
  200 CONTINUE
C************************************phi integration angles
      DQ=(Q2-Q1)/FLOAT(NDIVP)
      DO 201 I=1,NDIVP+1
        Q(I)=(I-1)*DQ
  201 CONTINUE
      SUM=0.
      SUMNONE=0.
      UMAX=0.
      UMAXNONE=0.
      EMAX=0.
      EMAXNONE=0.
C*************************V phi integration of rad intensity * beam solid angle V
      DO 300 JJ=1,NDIVP
        P1=DQ/2.
        P2=(Q(JJ+1)+Q(JJ))/2.
C************************************V phi integration determining Escattered V
C                                   V   within each interval of phi, NDIVP    V
        DO 300 J=1,NPHI
          PHR=P1*X(J)+P2
C*****************V theta integration of rad intensity * beam solid angle V
          DO 301 II=1,NDIVT
            T1=DS/2.
            T2=(S(II+1)+S(II))/2.
C**************************V theta integration determining Escattered V
C                          V   within each interval of theta, NDIVT   V
            DO 301 I=1,NPHI
              THR=T1*X(I)+T2
              IF(THR.LT.PI) GO TO 302
              THR=(BK-THR)
              PHR=PHR+PI
  302         CONTINUE
C****************************************************Beam Solid Angle
              STRA=SIN(THR)*A(I)*A(J)
              KJ=(JJ-1)*NPHI+J
              KI=(II-1)*NPHI+I
              THRR(KJ,KI)=THR*180/PI
```

```
            PHRR(KJ)=PHR*180/PI
            SNP=SIN(PHR)
            CNP=COS(PHR)
C*************************************************************************************
C    _    _                  _                                  _   _     _
C   |      |                 |                                 | |        |
C   |   s  |                 | _ t,theta          _ phi,theta  | |  _ t   |
C   | E    |        -jkr  MHI | R                  R            | |  I     |
C   |  t   |    -jhe      --- |  n                  n           | |   n    | jnphi
C   |      | =  _____      \  |                                 | |        |e
C   |   s  |    4pir      /__ | _ t,phi            _ phi,phi    | |  _ phi|
C   | E    |              n=1 | R                  R            | |  I     |
C   |  phi|                   |  n                  n           | |   n    |
C   |_   _|                   |_                               _| |_     _|
CV*********************************see page 25. H,E, and Combined Field********V
C                        -jkr
C                    -jhe
C Leading Term       ______   : eta (h) is cancelled by its inverse in CT (I),
C                     4pir      r dependence is ignored, and the whole Term is
C                               scaled by 2pi for gauss quadrature integration
C
C Leading Term is reduced to (-j/4pi)/2pi
C*************************************************************************************V
C           LEADTERM=UC/BK
            LEADTERM=UC
            ET=(0,0)
            EP=(0,0)
            ET1=(0.,0.)
            EP1=(0.,0.)
            ET2=(0.,0.)
            EP2=(0.,0.)
C*************************************************************************************V
C      MHI
C      ---
C      \
C      /__
C      n=1
C*************************************************************************************V
            DO 305 M=1,MHI
              NM=M-1
C************************************************************** jnphi
C                                                              e
              EXP1=CEXP(CMPLX(0.,FLOAT(NM)*PHR))
              EXP2=CONJG(EXP1)
C*************determine plane wave excitation vector in far field, R
              CALL PLANE(NM,NM,NP,NT,RH,ZH,XT,AT,THR,R)
              NTOP1=MODES-NM
              NTOP2=NBLOCK-(NTOP1+1)
              NS2=NTOP1*N
              NS1=NTOP2*N
C**************************************************************
```

```
C       | _ t,theta| | _ t |  jnphi        | _ t,phi   | | _ t |  jnphi
C       | R          |.| I   | e       ,and  | R          |.| I   | e
C       |  n         | |  n  |               |  n         | |  n  |
C********************************************************************
                DO 306 L=1,MT
C************************************************positive mode +n
                  ET1=ET1+R(L)*CT(L+NS1)*EXP1
                  EP1=EP1+R(L+N)*CT(L+NS1)*EXP1
C************************************************negative mode -n
                  IF(NM.EQ.0) GO TO 306
                  ET1=ET1+R(L)*CT(L+NS2)*EXP2
                  EP1=EP1-R(L+N)*CT(L+NS2)*EXP2
306             CONTINUE
C********************************************************************
C  | _ phi,theta| | _ t |  jnphi        | _ phi,phi   | | _ t |  jnphi
C  | R          |.| I   | e       ,and  | R            |.| I   | e
C  |  n         | |  n  |               |  n           | |  n  |
C********************************************************************
                DO 307 L=1,MP
C************************************************positive mode +n
                  ET2=ET2+R(L+MT)*CT(L+NS1+MT)*EXP1
                  EP2=EP2+R(L+MT+N)*CT(L+NS1+MT)*EXP1
C************************************************negative mode -n
                  IF(NM.EQ.0) GO TO 307
                  ET2=ET2-R(L+MT)*CT(L+NS2+MT)*EXP2
                  EP2=EP2+R(L+MT+N)*CT(L+NS2+MT)*EXP2
307             CONTINUE
c                ET=ET+LEADTERM*(ET1+ET2)
c                EP=EP+LEADTERM*(EP1+EP2)
305           CONTINUE
C************************************************^ end mode summation ^
              ETT=LEADTERM*(ET1+ET2)
              EPT=LEADTERM*(EP1+EP2)
*******************************************add in antenna contribution
C FEED CONTRIBUTION IN THE FAR FIELD IS ETF,EPF
C****************************************************BESSJ1.
C USE CLOSED FORM FEED EXPRESSION FROM SUBROUTINE ANTFF IF
C IFF=0; OTHERWISE USE BRUTE FORCE EVALUATION FROM CIRCRTP
C
              ROBS=1000.
              IF(IFF.EQ.0) THEN
               CALL ANTFF(THR,PHR,THS,PHS,ARAD,ETF,EPF)
C        write(6,*)' ET = ',CABS(ET),' ETF = ',CABS(ETF)
              ELSE
               RHB=ROBS*SIN(THR)
               ZHB=ROBS*COS(THR)
               CALL CIRCRTP(CNPHI,XP,AP,ARAD,THS,PHS,
     &                      PHR,RHB,ZHB,CIRCR,CIRCT,CIRCP)
C REMOVE THE 1/R DEPENDENCE BECAUSE EXP(-jkR)/R IS OMITTED IN
C THE SCATTERED FIELDS ET AND EP
               ETF=CIRCT*ROBS
```

```
             EPF=CIRCP*ROBS
C       write(6,*)' ET = ',CABS(ET),' ETF(ROBS) = ',CABS(ETF)
            ENDIF
C***********************************************total E theta and E phi
C       write(6,*)'THR(',jj,j,ii,i,NM,') = ',180*THR/pi
            ETCOMB=ETT+ETF
            EPCOMB=EPT+EPF
            EMAG=SQRT(CABS(ETCOMB)**2+CABS(EPCOMB)**2)
            EMAGNONE=SQRT(CABS(ETF)**2+CABS(EPF)**2)
            IF(EMAG.GT.EMAX)THEN
              EMAX=EMAG
              EMAXNONE=EMAGNONE
              WRITE(6,*)' EMAG(',J,I,') = ',EMAG
            END IF
C*****************************************determine max field intensity
            UMAG=EMAG**2
            UMAGNONE=EMAGNONE**2
            IF(UMAG.GT.UMAX)THEN
             UMAX=UMAG
             THETAMAX=180*THR/PI
             PHIMAX=180*PHR/PI
C             WRITE(6,*)' UMAX = ',UMAX
C             WRITE(6,*)' THETAMAX = ',THETAMAX
C             WRITE(6,*)' PHIMAX = ',PHIMAX
           END IF
           IF(UMAGNONE.GT.UMAXNONE)THEN
            UMAXNONE=UMAGNONE
           END IF
C***********************multiply integration by beam solid angle (STRA)
           SUM=SUM+UMAG*STRA
           SUMNONE=SUMNONE+UMAGNONE*STRA
301      CONTINUE
C**************************************************^ end theta integration ^
300   CONTINUE
C*********************************************************^ end phi integration ^
C*************************scale each summation by (b-a)/2 for Gauss integration
      PRAD=T1*P1*SUM
      PRADNONE=T1*P1*SUMNONE
            WRITE(6,*)'PRAD = ',PRAD
            WRITE(6,*)'PRAD NO RADOME = ',PRADNONE
C*******************************************************************************
C   End Integration over closed surface radiation intensity * beam solid angle
C*******************************************************************************
C*******************************************************************compute gain
      GAIN=4*PI*UMAX/PRAD
      GAINNONE=4*PI*UMAXNONE/PRADNONE
      GDB=10.*alog10(GAIN)
      GDBNONE=10.*alog10(GAINNONE)
      WRITE(6,*)'GAIN',NDIVT,' = ',GAIN,' IN DB = ',GDB
      WRITE(6,*)'GAINNONE = ',GAINNONE,' IN DB = ',GDBNONE
      WRITE(7,*)'NDIVT(',NDIVT,')=',NDIVT,'; NDIVP(',NDIVP,')=',NDIVP
```

```
      WRITE(7,2003)'GAIN(',NDIVT,')=',GAIN,';GAINDB(',NDIVT,')=',GDB
      WRITE(7,2003)'GAINNONE(',NDIVT,') = ',GAINNONE,
     &';GAINNONEDB(',NDIVT,')  = ',GDBNONE
2003  FORMAT(/(A,I3,A,F12.5,A,I3,A,F10.5))
C500   continue
      STOP
 999  end
C*************************************************************END OF MAIN PROGRAM
C*****************************************************SUBROUTINE PLANE
C REFERENCE: TECHNICAL REPORT TR-80-1 ;(pages 57-64)
      SUBROUTINE PLANE(M1,M2,NP,NT,RH,ZH,XT,AT,THR,R)
C
C PLANE WAVE EXCITATION VECTOR IN THE FAR FIELD. FROM MAUTZ AND HARRINGTON.
C MODIFIED TO DO ONLY ONE ANGLE AND FREQUENCY PER CALL.
C ** NEW BESSEL FUNCTION ROUTINE FROM NUM. RECIPES **
C
      COMPLEX R(1600),U,U1,UA,UB,FA(1500),FB(1500),F2A,F2B,F1A,F1B
      COMPLEX U2,U3,U4,U5
      DIMENSION RH(400),ZH(400),XT(4),AT(4),R2(4),Z2(4)
      MP=NP-1
      MT=MP-1
      N=MT+MP
      N2=2*N
      CC=COS(THR)
      SS=SIN(THR)
      U=(0.,1.)
      U1=3.141593*U**M1
      M3=M1+1
      M4=M2+3
      IF(M1.EQ.0) M3=2
      M5=M1+2
      M6=M2+2
      DO 12 IP=1,MP
      K2=IP
      I=IP+1
      DR=.5*(RH(I)-RH(IP))
      DZ=.5*(ZH(I)-ZH(IP))
      D1=SQRT(DR*DR+DZ*DZ)
      R1=.25*(RH(I)+RH(IP))
      IF(R1.EQ.0.) R1=1.
      Z1=.5*(ZH(I)+ZH(IP))
      DR=.5*DR
      D2=DR/R1
      DO 13 L=1,NT
      R2(L)=R1+DR*XT(L)
      Z2(L)=Z1+DZ*XT(L)
   13 CONTINUE
      D3=DR*CC
      D4=-DZ*SS
      D5=D1*CC
      DO 23 M=M3,M4
```

```
      FA(M)=0.
      FB(M)=0.
   23 CONTINUE
      DO 15 L=1,NT
      X=SS*R2(L)*2.
      ARG=Z2(L)*CC
      UA=AT(L)*CMPLX(COS(ARG),SIN(ARG))
      UB=XT(L)*UA
C THIS LINE REPLACES HARRINGTON'S BLOCK
      DO 25 M=M3,M4
      BES=BESSJ(M-2,X)
      FA(M)=BES*UA+FA(M)
      FB(M)=BES*UB+FB(M)
   25 CONTINUE
   15 CONTINUE
      IF(M1.NE.0) GO TO 26
      FA(1)=-FA(3)
      FB(1)=-FB(3)
   26 UA=U1
      DO 27 M=M5,M6
      M7=M-1
      M8=M+1
      F2A=UA*(FA(M8)+FA(M7))
      F2B=UA*(FB(M8)+FB(M7))
      UB=U*UA
      F1A=UB*(FA(M8)-FA(M7))
      F1B=UB*(FB(M8)-FB(M7))
      U4=D4*UA
      U2=D3*F1A+U4*FA(M)
      U3=D3*F1B+U4*FB(M)
      U4=DR*F2A
      U5=DR*F2B
      K1=K2-1
      K4=K1+N
      K5=K2+N
      R(K2+MT)=-D5*(F2A+D2*F2B)
      R(K5+MT)=D1*(F1A+D2*F1B)
      IF(IP.EQ.1) GO TO 21
      R(K1)=R(K1)+U2-U3
      R(K4)=R(K4)+U4-U5
      IF(IP.EQ.MP) GO TO 22
   21 R(K2)=U2+U3
      R(K5)=U4+U5
   22 K2=K2+N2
      UA=UB
   27 CONTINUE
   12 CONTINUE
      RETURN
      END
C*******************************************SUBROUTINE ANTFF
      SUBROUTINE ANTFF(THETA,PHI,THETAS,PHIS,ARAD,ETF,EPF)
```

```
C COMPUTES CLOSED FORM ANTENNA PATTERN IN THE FAR FIELD.
C ETF AND EPF ARE RETURNED. ANGLES ARE IN RADIANS.
C PRESENT ENTRY IS FOR:
C
C UNIFORM ILLUMINATION SCANNED TO A DIRECTION (THETAS,PHIS)
C
      COMPLEX ETF,EPF,JK,SCL,EB
      JK=(0.0,6.283185307)
      BK=6.283185307
      PI=BK/2.
C SET RADIATION PATTERN TO ZERO IN THE REAR HEMISPHERE
      IF(COS(THETA).LT.0.) THEN
        EB=(0.,0.)
      ELSE
        BB=SIN(THETA)*SIN(PHI)-SIN(THETAS)*SIN(PHIS)
        AA=SIN(THETA)*COS(PHI)-SIN(THETAS)*COS(PHIS)
        ARG=BK*ARAD*SQRT(AA**2+BB**2)
        SCL=-(0.,1.)*2.*PI**2
        IF(ABS(ARG).LT.1.E-5)THEN
          EB=SCL/2.
          ELSE
          EB=SCL*(BESSJ1(ARG)/ARG)
        ENDIF
      ENDIF
C EB IS THE X COMPONENT; GET ETHETA (ETF) AND EPHI (EPF)
C ASSUME AN ELEMENT FACTOR, ELMT
       ELMT=SQRT(ABS(COS(THETA)))
       ETF=EB*COS(THETA)*COS(PHI)*ELMT
       EPF=-EB*SIN(PHI)*ELMT
      RETURN
      END
C***********************************************BESSEL FUNCTIONS
      FUNCTION BESSJ(N,X)
C RETURNS THE BESSEL FUNCTION B OF ORDER N (>1) AND REAL
C ARGUMENT X.
      PARAMETER (IACC=40,BIGNO=1.E10,BIGNI=1.E-10)
      IF(N.EQ.0) THEN
C IF N=0 CALL BESSJ0
        BESSJ=BESSJ0(X)
       ELSE IF(N.EQ.1) THEN
C IF N=1 CALL BESSJ1
        BESSJ=BESSJ1(X)
       ELSE
C IF N>1 USE RECURSION
      BESSJ=0.
      IF(X.NE.0.) THEN
      TOX=2./X
      IF(X.GT.FLOAT(N)) THEN
      BJM=BESSJ0(X)
      BJ=BESSJ1(X)
      DO 11 J=1,N-1
```

```
      BJP=J*TOX*BJ-BJM
      BJM=BJ
      BJ=BJP
   11 CONTINUE
      BESSJ=BJ
      ELSE
      M=2*((N+INT(SQRT(FLOAT(IACC*N))))/2)
      BESSJ=0.
      JSUM=0.
      SUM=0.
      BJP=0.
      BJ=1.
      DO 12 J=M,1,-1
      BJM=J*TOX*BJ-BJP
      BJP=BJ
      BJ=BJM
      IF(ABS(BJ).GT.BIGNO) THEN
      BJ=BJ*BIGNI
      BJP=BJP*BIGNI
      BESSJ=BESSJ*BIGNI
      SUM=SUM*BIGNI
      ENDIF
      IF(JSUM.NE.0) SUM=SUM+BJ
      JSUM=1-JSUM
      IF(J.EQ.N) BESSJ=BJP
   12 CONTINUE
      SUM=2.*SUM-BJ
      BESSJ=BESSJ/SUM
      ENDIF
      ENDIF
      ENDIF
      RETURN
      END
      FUNCTION BESSJ0(X)
C
C BESSEL FUNCTION OF 0 ORDER, REAL ARGUMENT X
C (SEE 'NUMERICAL RECIPES', P.172)
C
      REAL*8 Y,P1,P2,P3,P4,P5,Q1,Q2,Q3,Q4,Q5,R1,R2,R3,R4,R5,R6,
     * S1,S2,S3,S4,S5,S6
      DATA P1,P2,P3,P4,P5/1.D0,-.109862827D-2,.2734510407D-4,
     * -.2073370639D-5,.2093887211D-6/
      DATA Q1,Q2,Q3,Q4,Q5/-.1562499995D-1,.1430488765D-3,
     * -.6911147651D-5,.7621095161D-6,-.934945152D-7/
      DATA R1,R2,R3,R4,R5,R6/57568490574.D0,-13362590354.D0,
     * 651619640.7D0,-11214424.18D0,77392.33017D0,-184.9052456D0/
      DATA S1,S2,S3,S4,S5,S6/57568490411.D0,1029532985.D0,
     * 9494680.718D0,59272.64853D0,267.8532712D0,1.D0/
      IF(X.EQ.0.) THEN
       BESSJ0=1.
      ELSE IF(ABS(X).LT.8.) THEN
```

```
      Y=X**2
      BESSJ0=(R1+Y*(R2+Y*(R3+Y*(R4+Y*(R5+Y*R6)))))/
     *    (S1+Y*(S2+Y*(S3+Y*(S4+Y*(S5+Y*S6)))))
      ELSE
      AX=ABS(X)
      Z=8./AX
      Y=Z**2
      XX=AX-.785398164
      BESSJ0=SQRT(.636619772/AX)*(COS(XX)*(P1+Y*(P2+Y*(P3+
     *    Y*(P4+Y*P5))))-Z*SIN(XX)*(Q1+Y*(Q2+Y*(Q3+
     *    Y*(Q4+Y*Q5)))))
      ENDIF
      RETURN
      END
      FUNCTION BESSJ1(X)
C
C BESSEL FUNCTION B OF ORDER 1, REAL ARGUMENT X
C (SEE 'NUMERICAL RECIPES', P.173)
C
      REAL*8 Y,P1,P2,P3,P4,P5,Q1,Q2,Q3,Q4,Q5,R1,R2,R3,R4,R5,R6,
     * S1,S2,S3,S4,S5,S6
      DATA P1,P2,P3,P4,P5/1.D0,.183105D-2,-.3516396496D-4,
     * .2457520174D-5,-.20337019D-6/
      DATA Q1,Q2,Q3,Q4,Q5/.04687499995D0,-.2002690873D-3,
     * .8449199096D-5,-.99228987D-6,.105787412D-6/
      DATA R1,R2,R3,R4,R5,R6/72362614232.D0,-7895059235.D0,
     * 242396853.1D0,-2972611.439D0,15704.4826D0,-30.16036606D0/
      DATA S1,S2,S3,S4,S5,S6/144725228442.D0,2300535178.D0,
     * 18583304.74D0,99447.43394D0,376.9991397D0,1.D0/
      IF(X.EQ.0.) THEN
       BESSJ1=0.
      ELSE IF(ABS(X).LT.8.) THEN
      Y=X**2
      BESSJ1=X*(R1+Y*(R2+Y*(R3+Y*(R4+Y*(R5+Y*R6)))))/
     *    (S1+Y*(S2+Y*(S3+Y*(S4+Y*(S5+Y*S6)))))
      ELSE
      AX=ABS(X)
      Z=8./AX
      Y=Z**2
      XX=AX-2.356194491
      BESSJ1=SQRT(.636619772/AX)*(COS(XX)*(P1+Y*(P2+Y*(P3+
     *    Y*(P4+Y*P5))))-Z*SIN(XX)*(Q1+Y*(Q2+Y*(Q3+
     *    Y*(Q4+Y*Q5)))))*SIGN(1.,X)
      ENDIF
      RETURN
      END


C**************************************************SUBROUTINE CIRCRTP
C SUBROUTINE    :       CIRCRTP
C DATE          :       1 OCTOBER 1991
```

```
C REVISED      :        26 February 1992
C PROGRAMMER   :        R.M. FRANCIS
C 2ND REVISION :        27 October 1992
C PROGRAMMER   :        K. A. KLOPP
C
C COMMENTS :  THIS SUBROUTINE WILL RIGOROUSLY CALCULATE THE ELECTRIC FIELD
C FOR A CIRCULAR APERTURE. THE FIELD IS CALCULATED AT THE COORDINATES
C SPECIFIED BY RH(.) AND ZH(.)  THE APERTURE IS LOCATED AT Z = 0,
C AND SCANNED TO A DIRECTION (THS,PHS). THE SUBROUTINE OGIVE IS THE
C SOURCE OF THE GEOMETRIC DATA REQUIRED BY CIRC TO PERFORM COMPUTATIONS.
C ALL PHYSICAL DIMENSIONS ARE NORMALIZED TO WAVELENGTH.
C
C*****************************************************************************
C                      +ant   +ant                                            *
C                       _      _                                              *
C    i                 \      \      x1                                       *
C   E  = -jn/(4*pi*k)  /_     /_    e (.......)*TAPER(rp)                     *
C                      x=0    y=0                                             *
C                                                                             *
C*****************************************************************************
      SUBROUTINE CIRCRTP(CNPHI,XP,AP,ARAD,THS,PHS,
     *                           PHI,RZ,Z,CIRCR,CIRCT,CIRCP)
      INTEGER CNPHI
      DIMENSION XP(100),AP(100)
      REAL K
      COMPLEX J,JK,G1,G2,X1,CON,CC,SUMX,SUMY,SUMZ
      COMPLEX CIRCR,CIRCT,CIRCP
      PI=3.141592654  .
      ETA=120.*PI
C SET FIELD COMPONENTS TO ZERO IN THE REAR HEMISPHERE
      CIRCR=(0.,0.)
      CIRCT=(0.,0.)
      CIRCP=(0.,0.)
C Z OF THE OBSERVATION POINT IS >= 0 IN THE FORWARD HEMISPHERE
      IF(Z.GE.0.) THEN
      SUMX=(0.0,0.0)
      SUMY=(0.0,0.0)
      SUMZ=(0.0,0.0)
      K=6.283185307
      J = (0.0,1.0)
      JK= (0.0,6.283185307)
      CON=-J/(4.*PI)
C OMIT THE FACTOR OF ETA SINCE IT IS NOT INCLUDED IN SUBROUTINE ZMAT.
C ARAD**2 IS FOR SCALING OF GAUSSIAN INTEGRATION.  READJUST SCALING
C TO AGREE WITH OLD SUBROUTINE SPHERE (ARBITRARY)
      CC= CON
C OUTER  LOOP: INTEGRATE OVER X, -ARAD < X < ARAD.
C INNER  LOOP: INTEGRATE OVER Y, -ARAD < Y < ARAD.
C EVALUATE ANTENNA FIELD AT RZ,Z:
      XO=RZ*COS(PHI)
      YO=RZ*SIN(PHI)
```

```
          STS=SIN(-THS)
          S=RZ/SQRT(RZ**2+Z**2)
          C=Z/SQRT(RZ**2+Z**2)
C INTEGRATE IN X,Y COORDINATES
            DO 50 M=1,CNPHI
               X=ARAD*XP(M)
              DO 60 N=1,CNPHI
               Y=ARAD*XP(N)
         RP=SQRT(X**2+Y**2)
C SKIP INTEGRATION POINTS BEYOND THE ANTENNA RADIUS
        IF(RP.LE.ARAD) THEN
        PHIPRIME=ATAN2(Y,X+1.E-10)
        R=SQRT(((RZ-RP)**2+Z**2)+(4*RZ*RP*SIN((PHI-PHIPRIME)/2)**2))
        G1=(((K*R)**2)-1-(JK*R))/R**3
        G2=(3+(3*JK*R)-(K*R)**2)/R**5
        X1=JK*(RP*COS(PHIPRIME-PHS)*STS-R)
        Y1=XO-X
        X2=Y1**2
        Y2=YO-Y
           SUMX=SUMX+(CC*AP(M)*AP(N)*(G1+(X2*G2))*CEXP(X1))*TAPER(RP)
           SUMY=SUMY+(CC*AP(M)*AP(N)*Y1*Y2*G2*CEXP(X1))*TAPER(RP)
           SUMZ=SUMZ+(CC*AP(M)*AP(N)*Y1*Z*G2*CEXP(X1))*TAPER(RP)
        ENDIF
   60      CONTINUE
   50      CONTINUE
C ASSUME AN ELEMENT FACTOR, ELMT
        ELMT=SQRT(ABS(C))
        CIRCT=(C*COS(PHI)*SUMX+C*SIN(PHI)*SUMY-S*SUMZ)*ELMT
        CIRCR=(S*COS(PHI)*SUMX+S*SIN(PHI)*SUMY+C*SUMZ)*ELMT
        CIRCP=(-SIN(PHI)*SUMX+COS(PHI)*SUMY)*ELMT
        ENDIF
        RETURN
        END
C*******************************************FUNCTION TAPER
        FUNCTION TAPER(RHO)
C SPECIFY ANTENNA ILLUMINATION FUNCTION.  REAL FUNCTION OF
C RHO ONLY
        TAPER=1.
        RETURN
        END
```

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